Bank Regulation and Supervision: A Symbiotic Relationship*

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Abstract

Supervisory assessments such as stress-tests gauge banks' riskiness and allow regulators to impose bank-specific capital regulation. This can improve welfare. Yet, regulation based on noisy supervision can decrease welfare by mis-classifying banks, distorting incentives, and creating riskier banks. Regulation should not be bank-specific in such cases. When bank defaults are costlier, supervision should lower the probability that riskier banks go undetected, i.e., reduce false-negatives even if this leads to more false-positives. When the supervisor can conduct a more comprehensive but costlier assessment that optimally reduces both false-positive and false-negative rates, the regulator should make capital requirements more bank specific.

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1 Introduction

Supervision and regulation are the two main pillars of a typical banking industry oversight regime. Supervision involves assessing the safety of financial institutions by conducting exams (such as stress-tests), while regulation refers to setting rules (such as minimum capital-ratio requirements) under which banks operate. Since the great financial crisis in 2008, policymakers have increasingly relied on the joint use of supervision and regulation. Supervisory risk assessments generate information about banks' risk exposures that goes beyond what is usually found in standard financial reporting and disclosures by banks. In turn, this helps regulators better align capital regulation with individual banks' risk profiles.[1]

In principle, supervision based regulation can improve welfare. However, in practice, such welfare gains depend on the precision of supervisory tools. Several studies have shown that these tools, especially stress-tests, provide noisy assessments of banks' risk exposures [Plosser and Santos, 2018; Acharya et al., 2014]. Despite empirical evidence of such imprecision, the literature lacks a theoretical framework to think about how capital regulation must be set when supervision is known to be noisy, or to study the trade-offs associated with making supervision less noisy. For instance, the degree to which regulation should be calibrated based on supervisory results is not obvious. It is also not clear if supervision should be designed to minimize the chances of wrongly penalizing healthy banks (false positive rate) or that of failing to identify risky ones (false negative rate).

Our goal in this paper is to start filling these gaps. We develop a tractable model to first study the trade-offs associated with setting bank capital regulation on the basis of a potentially noisy supervisory assessment of banks' riskiness. We then use the model to assess the trade-offs in choosing the optimal bank-specific regulation while also adjusting

[1] In the U.S., capital surcharges (among other requirements) are determined on the basis of stress-test results. Likewise, in the euro area, stress-tests conducted by the European Banking Authority (EBA) are a crucial input into such bank specific requirements as capital planning, reporting, and governance.

the supervisory design. We consider two cases: (a) where the supervisor cannot reduce the false positive rate without worsening the false negative rate (and vice versa) and (b) where the supervisor can incur a cost to simultaneously reduce both error rates. Our main contribution is to show how supervision and regulation interact, and to draw insights regarding the jointly optimal supervision based regulation given strategic behavior by banks.

The key players in our model are a banker, a supervisor, and a regulator. The banker runs a bank that takes deposits and invests in a risky project. The return on the project can be *high* or *low*, depending on the banker's *type*, which in turn depends on the effort it exerts ex-ante. A mis-priced deposit insurance combined with limited liability induce the bank to over-borrow relative to the social optimal. In turn, this rationalises a minimum capital ratio requirement and allows us to study the welfare implications of varying that requirement.[2]

We assume that the regulator cannot observe the bank's type, which means that it cannot impose bank-specific regulation. The supervisor, say by conducting a stress-test, can generate a potentially imprecise signal about the bank's type.[3] This enables the regulator to then impose on top of the baseline capital requirement a bank-specific surcharge depending on its performance in the stress-test. For example, a higher surcharge is imposed on banks deemed to be of the *low* type based on the supervisory assessment.[4]

We show that the regulator faces a trade-off in setting a supervision-based capital surcharge. Supervision helps overcome (some) information frictions, which can improve

[2]A large related literature concerned with the issue of optimal capital regulation provides several rationales for capital-ratio requirements, such as fire-sale externalities [Kara and Ozsoy, 2020], implicit government guarantees [Nguyen, 2015], moral hazard issues [Christiano and Ikeda, 2016; Gertler and Kiyotaki, 2010], and household preference for safe and liquid assets [Begenau, 2020]. The approach in this paper is related to that of Kareken and Wallace [1978], Santos [2001], and Van den Heuvel [2008] who show that over-borrowing, led by mis-priced deposit insurance or otherwise, justifies capital regulation.

[3]Stress-tests are one of the several ways in which regulators can obtain a signal about specific characteristics of banks. There are, indeed, other micro-prudential and supervisory tools, such as onsite risk assessments, that may provide similar signals and thus be subject to similar trade-offs that we model in the context of stress-tests.

[4]For the sake of simplicity, we consider the supervisory assessment to be a binary classifier of banks. In this case, the Receiver Operating Characteristics or ROC curve captures the accuracy of the assessment. See Hanley and McNeil [1982], for example.

welfare. Yet, inaccuracies in supervision can lead to mis-classification of banks. This not only lowers welfare directly, but also indirectly as a capital surcharge based on noisy supervision can distort banks' ex-ante incentives and induce them to become more risky. This is because stricter regulation on an otherwise high-type bank due to an incorrect supervisory assessment entails an opportunity cost. In turn, this cost reduces banks' ex-ante incentives to exert effort towards becoming a high-type bank. At the same time, a lower likelihood that a low-type bank is not identified as low-type and is therefore not penalized via a surcharge also lowers incentives to exert ex-ante effort. Thus, in contrast to conventional wisdom, we show that in the presence of information frictions, higher capital requirements may lead to riskier banks.[5]

We find that the optimal surcharge is zero when the chances of mis-classifying banks is high. When the assessment is sufficiently accurate, the optimal surcharge increases with accuracy as it has a strong disciplining effect i.e. eliciting greater effort from the bank.[6] When bank default is more costly, such as in the case of too-big-to-fail banks, not only is the optimal baseline requirement stricter, the optimal capital surcharge is also higher for a given level of accuracy.

Next, we interact more closely the problem of the supervisor with that of the regulator. Specifically, we enable the supervisor to choose the degree of supervisory accuracy, and

[5]How banks respond to capital requirements is a key element of our analysis, one that complements previous studies that have analysed the relationship between capital requirements and bank risk. For instance, Koehn and Santomero [1980] show that tighter capital requirements can lead portfolio managers to become more risky. In follow-up research, Kim and Santomero [1988] and Rochet [1992] show that this result disappears when risk-weights used to compute capital requirements are consistent with asset quality. We overturn this result, basing our analysis on the fact that risk assessment is inherently noisy, and showing that in such cases capital requirements can lead to adverse incentives. While this insight resonates with that in Prescott [2004] where poorly executed supervisory audits induce banks to disclose information strategically, or in Gale et al. [2010] where higher capital can force banks to take more risk to achieve the required rate of return, the underlying mechanism in our paper is distinct. We show that a moral hazard issue arises as the opportunity cost of tighter regulation is greater for a high-type bank.

[6]Our paper formalises the intuition James Bullard (President of the Federal Reserve Bank of St. Louis) had in the context of quantitative easing: "while state-contingent policies are generally desirable, they work well when the states on which the policy is contingent are known" [Bullard, 2013]. Relatedly, our paper supports the remarks made by Mark Zelmer (Deputy Superintendent, OSFI Canada) in 2013 in the context of risk-sensitivity of capital requirements [Zelmer, 2013].

study the jointly optimal supervisory accuracy and capital surcharge. We consider two cases.

First, we assume that it is not possible to improve one error rate (say false-positive) without worsening the other error rate (i.e. false-negative). This is typically the case when the supervisor cannot improve the design of the test (say because it is prohibitively costly to do so) and can only adjust the (signal) cutoff below which a bank is considered to be of the low type. This is akin to moving along the ROC curve of the supervisory assessment (recall footnote 4). We show in this case that as the cost of bank default rises, the supervisor chooses a lower false-negative rate, even though this means that the false-positive rate is higher. Simply put, the supervisor makes passing the test *generally* harder. For the regulator, this response goes hand-in-hand with a higher capital surcharge. Therefore, this jointly optimal supervision-based regulation strategy ensures that banks that fail the test are better capitalised and are less likely to impose significant costs on the economy in the event of default.

Second, we assume that the supervisor can alter the design of the risk assessment by incurring a social cost – say due to higher organisational or logistic burden on itself and also on the banks (see for e.g. Eisenbach et al. [2016]) – and reduce both false-positive and false-negative rates simultaneously. This is akin to increasing the Area Under the ROC curve (AUROC, recall footnote 4). We show that as the cost of improving assessment accuracy becomes smaller, the supervisor optimally makes passing the test generally easier while the regulator increases the surcharge. Intuitively, making the test easier to pass leads to a lower false-positive rate but not a higher false-negative rate precisely because the assessment is overall more accurate. And because it is more certain that banks that fail the supervisory assessment are the low-type ones (higher true-positive rate), a larger surcharge is more effective in making banks safer ex-post and creates fewer adverse incentives.

Disclosure of the results of a supervisory assessment is another key element of banking oversight. In principle, such disclosures can entail a disciplinary effect on banks via the role of investors (e.g., Pillar 3 in Basel III). Indeed, we show that when stress-tests are sufficiently accurate in identifying bank types, disclosures improve market discipline and facilitate the use of capital surcharges. However, when stress-tests are less accurate, disclosures can amplify adverse incentives, induce greater risk-taking by banks, and thus place further limits on the use of surcharges.

To illustrate our analytical results, we calibrate the parameters of the model using data on twenty major economies. Numerical computations allow us to fully characterise the relationship between supervisory accuracy and optimal surcharge, and also conduct various counterfactual experiments that help confirm the theoretical predictions.

We conclude our discussion by alluding to potentially noisy assessments in the 2020 Dodd-Frank Act Stress Test in the U.S. that was conducted right before the Covid-19 crisis. We show that the cross-sectional dispersion in changes in banks' capital ratios in the stress-test was much higher than the observed changes, and that the two do not correlate. These observations point to substantial noise in stress-test results. As such, they are in line with similar findings in the literature, and further rationalise the importance of having a theoretical framework to inform supervision based regulation.[7]

Related literature

A large literature studies aspects of banking oversight, especially capital regulation and supervisory risk-assessment, but most papers analyse one aspect while taking the other as given.[8] In this paper, we strive to bridge these strands of the literature by studying

[7] An ideal appraisal of stress-testing would be one where the hypothetical stress scenario and the realised crisis are *identical*, but bank outcomes are *not*. While the Covid-19 crisis is not identical to the severely adverse scenario of the 2020 US stress-test, several of the key indicators that characterise a macroeconomic scenario (such as GDP, employment, and stock prices) experienced comparable shocks. As such, broad concordance in banks' relative performances in the test and in the current crisis is to be expected. See Section 5 for more details.

[8] While there is a large body of theoretical and empirical work on bank capital regulation that predates the global financial crisis, the literature on bank supervision is relatively new. Regulation and supervision are often used interchangeably in the literature and only recently have studies started to explore the impact of supervision (distinct from regulation) on various aspects of financial intermediation, including bank performance, credit supply, and bank risk taking (see Hirtle et al. [2020], Delis and Staikouras [2011],

how supervision and regulation interact. Our goal is to better understand the trade-offs a policymaker faces in jointly optimising the design of supervision and regulation when banks' can behave strategically.

A study of the interaction between supervision and regulation is important especially because potentially inaccurate supervision has implications for how the attendant regulation should be designed. Indeed, a growing literature provides empirical evidence on such inaccuracies – it shows that inaccuracies can stem from noisy bank-level inputs used in risk-assessment models [Ong et al., 2010], limits of internal risk models of banks [Leitner and Yilmaz, 2019; Plosser and Santos, 2018; Behn et al., 2016; Wu and Zhao, 2016], or limits of econometric models used by the supervisors to predict bank losses [Covas et al., 2014]. It could also be that these assessments do not fully take into account the endogenous reaction of banks to shocks [Braouezec and Wagalath, 2018].[9] For instance, Acharya et al. [2014] find that in the 2011 European stress-test, the assessment of banks' risk was not in line with their realized riskiness in the period after the stress-tests.[10] Despite acknowledgement of the noise inherent in supervisory risk assessments, the literature lacks a theoretical framework to think about the trade-offs policymakers face in choosing optimal bank-specific capital regulation and in designing an optimal supervisory assessment. Our paper makes progress in filling this gap.

In terms of the optimal design of supervisory risk assessments such as stress tests, the existing literature focuses mostly on the disclosure of test results [Goldstein and Sapra, 2014; Bouvard et al., 2015; Williams, 2017; Goldstein and Leitner, 2018; Orlov et al., 2018] or that of the regulatory models used in stress tests [Leitner and Williams, 2022].

and Bassett et al. [2015], for example.)

[9] Technical and computational glitches can also lead to noisy assessments. For example, in September 2020, the U.S. Federal Reserve Bank published corrections to its previously issued stress-test results [Fed, 2020].

[10] Philippon et al. [2017] find similar results for the 2014 stress-test conducted by the European Banking Authority. Relatedly, Frame et al. [2015] show that stress-tests conducted by the U.S. Office of Federal Housing Enterprise Oversight in the pre-GFC period failed to detect risks on the balance sheets of Fannie Mae and Freddie Mac. More generally, Berger et al. [2000] show that supervisory assessments are generally less accurate than market indicators in predicting banks' future performances.

A few recent studies have also explored the information acquisition aspect in the design of stress tests. For example, Parlatore and Philippon [2020] study the optimal design of stress scenarios that maximizes information acquisition about the multiple risk factors a bank is exposed to. Leitner and Yilmaz [2019] study the optimal monitoring intensity by a supervisor when banks can strategically choose to report the internal risk model they use to measure their risk exposures in a stress-test. Our paper contributes to this recent strand of the literature by studying a realistic constraint that supervisors face in designing accurate risk assessments – i.e. a higher probability of correctly identifying weak banks is often achieved at the expense of miss-classifying healthy banks more frequently. We assess when it is optimal to design a generally harder test even if it comes at the cost of some healthy banks failing the test.

Our paper also complements existing studies on optimal risk-sensitive or state contingent capital regulation. For instance Ahnert et al. [2020] and Morrison and White [2005] study the optimal capital requirement in a world with imperfect information about banks' types, while Marshall and Prescott [2001] and Lohmann [1992] study the case when there is uncertainty about the aggregate state of the economy. In particular, Ahnert et al. [2020] and Morrison and White [2005] show that a more accurate risk assessment precludes the need to impose a higher capital requirement as riskier banks are already selected out of the market because of better screening. By contrast, we consider risk assessment (i.e. supervision) and regulation as complements and not substitutes. We model a world where assessment is meant for identifying riskier banks while regulation is meant for preparing banks to better absorb those risks (without necessarily closing them). As such, our take-away is that regulation should be more bank-specific when supervisory accuracy is higher.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 discusses the issue of optimal regulation without supervision, while Section 4 studies optional regulation with supervision. Section 5 illustrates the analytical results using a calibrated version of the model. Section 6 provides suggestive evidence of noise in supervision using the stress-testing exercise conducted in the US right before the pandemic hit. Section 7 concludes.

2 Model

Our goal is to analyse the welfare and optimal policy implications of a banking oversight regime where capital regulation is imposed based on supervisory assessment of banks' risks. To this end, we develop a model where banks respond to incentives created by supervision as well as regulation. To be able to pursue a welfare analysis, we introduce an explicit rationale for policy intervention in the model. That is, while information frictions justify the use of supervision, a tendency for banks to assume inefficiently high leverage justifies regulation.

We consider an economy that lasts three periods (0, 1, and 2), and consists of a representative household, a banker whose decisions are socially inefficient and whose type is stochastic, a supervisor that cannot (fully) observe the bank's type, a regulator that sets capital requirements, and a government that runs a deposit insurance program.

Household The household is representative, and receives an unconditional income endowment $\bar{Y}$ on dates 1 and 2. On date-1, it decides how much to consume, c_1, and how much to deposit, d, in the bank.[11] Deposits are risk-free, and pay a gross return of R on date-2.

Banker The banker has a capital endowment of k on date-1. It runs a bank that issues deposits d to invest $k + d$ in a risky project that pays $\psi g(k + d)$ on date-2. $g(.)$ is a decreasing returns to scale (DRS) return function. ψ is an investment shock whose density f_s depends on the banker's type s on date-1, which can be high (H) or low (L). Specifically,

[11] A time subscript is used only for those quantities that are relevant on multiple dates. For instance, since d is only chosen once, on date-1, a time subscript is omitted.

we assume that while both types face the same standard deviation of ψ, namely σ, the high-type bank has a higher expected return, $\mu_H > \mu_L$, so that a high-type bank has a higher *risk-adjusted return.*The probability p with which the bank is of high-type depends on the effort e the banker exerts on date-0. The cost of exerting effort is $\zeta(e)$. The bank learns its type on date-1.

The bank's deposit liabilities on date-2 equal Rd, and thus the net cash-flow n equals $\psi g(k+d) - Rd$. When ψ is sufficiently high and the bank is solvent, the entire cash-flow is paid as dividends to the banker. However, when ψ is low enough so that the cash-flow is negative, the bank defaults and banker receives null. We assume that the banker only consumes on date-2, and that it has limited liability, so that it cannot be asked for additional capital to rescue a failing bank. Instead, the government takes the bank into receivership.

We assume that bank default entails a social cost.[12] Specifically, once a bank defaults, the recovery value of its assets is less that a hundred percent. This cost – denoted as Δ – is borne by a deposit insurance program.

Government The government runs the deposit insurance program and ensures that depositors are fully protected against bank failure. When a bank fails, the government liquidates its assets, and covers any shortfall in the failed bank's liabilities. To fund the scheme, the government imposes a lumpsum tax T on the household. We assume that the insurance scheme is mis-priced – ie insensitive to the risks banks take.[13] This, as we prove later, leads to a social inefficiency.[14] The government runs a balanced budget.

[12] In practice, this cost can stem from, for instance, forced sale of a failed bank's assets, as well as due to resolution related expenses. It can be a major cost in the case of large banks (due to contagion or knock-on effects), when the resolution framework is not well functioning, or during a crisis when many banks are in insolvency at the same time.

[13] Typical reasons for a mis-priced deposit insurance include the inability of the insurer to observe banks' risk profiles or impose risk-sensitive premium payments. See Flannery et al. [2017] for elaboration.

[14] The reason for introducing an inefficiency in our model is to rationalise capital requirements. A mis-priced deposit insurance is not the only way to do so, but it is a relatively simple method that helps keep our model tractable. Another paper to have taken this route is Van den Heuvel [2008].

Regulator The regulator is benevolent, i.e. it sets the minimum capital requirement with the objective to maximise the social welfare. It cannot, however, observe the bank's type on date-1.[15] In the baseline setup, therefore, it must announce a requirement $\mathcal{X}$ on date-0 that banks of either type must satisfy on date-1.

Supervisor The supervisor has access to a risk assessment tool that can produce valuable information about banks' types. This helps the regulator align the baseline capital requirement to the banks' types. For concreteness, we consider the supervisory tool to be a stress-test (another example of a risk assessment tool is onsite examination). We assume that the stress-test is a binary classifier. It produces a noisy signal about the bank's type. Then, based on the signal and a cutoff, the supervisor determines whether the bank has passed or failed the test, and accordingly deems it to be of the high or low type respectively (see Figure 1 for the timeline).[16] We assume that the probability that a high-type (low-type) bank passes the test is q_H (q_L).

The accuracy of the stress-test is fully captured by the tuple (q_H, q_L). Note that $(1-q_H)$ denotes the 'false positive' or Type-I error rate (high-type bank fails the test), while q_L is the 'false negative' or Type-II error rate (low-type bank passes the test). As such, in the full information case, i.e. when the signal perfectly identifies the type of the bank, $q_H = 1$ and $q_L = 0$.

The regulator uses the outcome of the supervisory exercise to adjust the baseline capital requirement $\mathcal{X}$. A bank that passes the stress-test is allowed to operate at the baseline requirement $\mathcal{X}$, whereas a failed bank faces a surcharge $x \geq 0$ on top of $\mathcal{X}$.[17]

[15] In reality, regulators do have some knowledge about banks' characteristics (such as via regulatory filings). We assume that a bank's type captures the *un-observable* features of the bank. Furthermore, we assume that the bank cannot credibly communicate its type to the regulator.

[16] In practice, most jurisdictions no longer assign an official pass or fail grade to the banks. Yet, this can still be inferred from whether its capital ratio under stress is below the minimum requirement.

[17] In line with stress-testing in practice, such as in the U.S. or Euro Area, we do not allow a capital relief for banks that do well in the test. Even if we were to allow for *negative* surcharges for such banks, we do not expect our qualitative findings to change because negative surcharges would be subject to the same regulatory trade-offs as the positive surcharges as long as stress-test are not perfect in identifying low and high type banks.

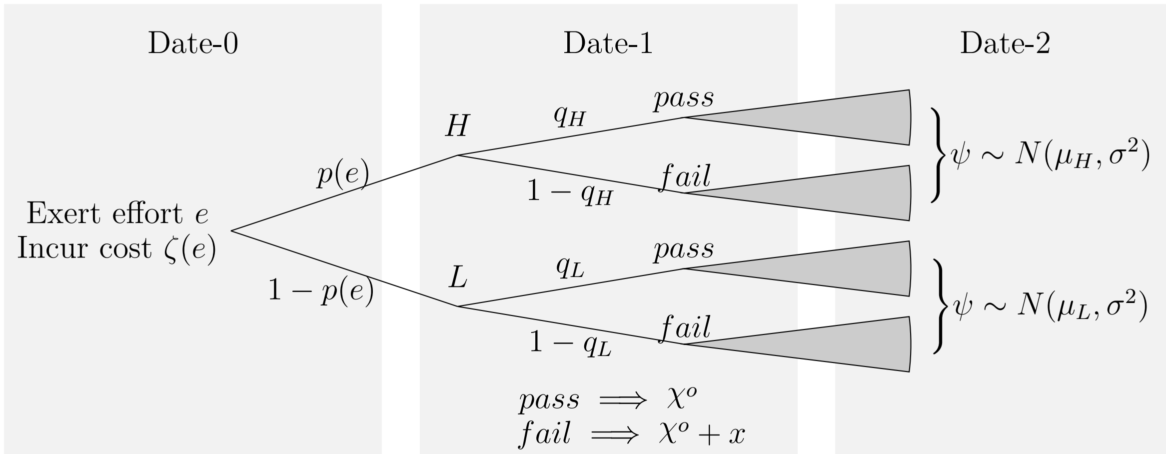

Figure 1: The timeline of events when there is information asymmetry about the bank's type, and stress tests serves as a tool to (partially) mitigate information frictions.

We analyse the joint problem of the regulator and the supervisor in steps. First we study the determination of optimal regulation – that is to find the optimal surcharge x for a given supervisory accuracy (q_H, q_L). Then we allow the supervisor to be able to choose (q_H, q_L) while being constrained to move along the receiver operating characteristics (ROC) curve of the stress-test, which means that improving one error rate necessarily worsens the other. Essentially, the supervisor shifts the signal cutoff η^c that pins down $q_H = 1 - Q_H(\eta^c)$ and $q_L = 1 - Q_L(\eta^c)$ for a given pair of stress-test generated signal distributions Q_H and (Q_L) for high and low-type banks respectively. Finally, to generalise further, we assume that the supervisor can incur a cost to redesign the stress-test (say make it more comprehensive) and improve both the false positive and negative error rates at the same time. This entails increasing the area under the ROC curve of the stress-test – that is its AUROC, a scalar metric that captures the overall performance of a binary classifier (Hanley and McNeil [1982]).

2.1 Competitive equilibrium

We now formally setup the problem statements of the agents in the economy. We focus on the case when there is no supervision (which means all banks face the same regulation).

The household chooses d on date-1 to maximize its expected utility over dates 1 and 2, where β is the discount factor:

$$U = \max_{d} \;\; c_1 + \beta \mathbb{E} c_2 \quad s.t. \quad c_1 = \bar{Y} - d \quad and \quad c_2 = \bar{Y} + Rd - T. \tag{1}$$

The banker chooses e on date-0 which determines the probability of being an H-type on date-1:

$$[Date - 0]: \quad \max_{e} \quad -\zeta(e) + \beta\Big(p(e)V_H(\chi) + (1 - p(e))V_L(\chi)\Big). \tag{2}$$

where $V_s(\chi)$ is defined in equation (3). The bank of type $s \in \{H, L\}$ chooses d on date-1 to maximize the expected dividend it pays on date-2:

$$[Date - 1]: \quad V_s(\chi) = \max_{d} \quad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \big(\underbrace{\psi g(k+d) - Rd}_{n}\big) f_s(\psi) d\psi \quad s.t. \quad \frac{k}{\chi} \geq d. \tag{3}$$

The lower limit on the integral is the ψ cut-off – call it ψ_c – below which the bank fails (and no dividends are paid). χ is the minimum capital-ratio requirement. The government's budget constraint is as follows:

$$T(\psi) = \begin{cases} Rd - \psi g(k+d)(1 - \Delta) & \text{If the bank defaults i.e. } \psi \leq \frac{Rd}{g(k+d)} \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

We now assess the optimality conditions in the baseline economy. The first-order condition (FOC) of the bank's problem on date-0 is as follows:

$$-\zeta'(e) + \beta p'(e) \underbrace{\Big(V_H(\chi) - V_L(\chi)\Big)}_{\omega} = 0 \tag{5}$$

This shows that the effort the bank exerts depends on the *wedge*, say ω, between the value of being a high- as opposed to low-type on date-1. To see how the effort changes as the wedge increases – a result we will use throughout the paper – we take the total derivative of Equation 5 with respect to ω, and note Lemma 1:

$$-\zeta''(e)\frac{de}{d\omega} + \beta p''(e)\omega\frac{de}{d\omega} + \beta p'(e) = 0 \tag{6}$$

Lemma 1. *Assume that the marginal cost of exerting effort is increasing (i.e. $\zeta(.)$ is convex) or the marginal benefit of effort in terms of improving the probability of being a high-type bank is decreasing (i.e. $p(.)$ is concave), then the bank exerts more effort when the difference in the value of being a high type compared to a low type increases.*

Intuitively, as the relative value of being a high-type bank increases, the incentive to exert effort becomes larger.[18] Lemma 1 thus shows that the minimum requirement (χ) – via its impact on wedge ω – is a key determinant of a bank's effort choice on date-0. In turn, this effects feeds back into the regulator's choice of the optimal requirement.

As regards the date-1 FOCs, we have the following:

$$\text{Bank:} \qquad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi - \Lambda_s = 0 \tag{7}$$

$$\text{Household:} \qquad R = 1/\beta \tag{8}$$

Note here that $s \in \{H, L\}$, Λ_s is the Lagrange multiplier on the regulatory constraint, and that two of the three terms which arise from an application of the Leibniz rule to derive the bank's date-1 FOC are equal to zero. The system of FOCs (5), (7), (8), the government's budget constraint (4), and the expression for the value function (3) together characterise the competitive equilibrium of the model economy for a given minimum capital-ratio requirement χ.

[18] Lemma 1 is related to a similar result proven in Christiano and Ikeda [2016], but the channel through which regulation has an impact on the banker's effort is different.

3 Optimal Regulation without Supervision

We begin by establishing the rationale for regulation in the model economy. Such a rationale is needed for a welfare analysis – that is, to discuss the role that regulation could play in improving welfare in the competitive equilibrium and to then determine the optimal regulation. To this end, we prove in Lemma 2 below that the competitive equilibrium is inefficient.

Lemma 2. *The competitive equilibrium is constrained inefficient. The bank's capital ratio in the competitive equilibrium is smaller (leverage is higher) as compared to that in the constrained planner's allocation, i.e. in the second best.*[19] *Moreover, the inefficiency is greater when bank default is more costly.*

Proof. See Appendix A. ■

Intuitively, the inefficiency stems from limited liability and a mis-priced deposit insurance. Because of limited liability, the bank does not internalise the losses corresponding to the left tail of the distribution of ψ – the part that corresponds to default. And because of deposit insurance, the depositors do not charge a premium for risk of non-repayment of deposit proceeds post bank failure. The bank thus over-borrows. The planner, on the contrary, chooses the level of deposits taking into account the entire distribution of ψ. We refer to the wedge between planner's and banker's (privately) optimal decisions as the bank-failure inefficiency

The inefficiency means that the welfare in the competitive equilibrium is lower than in case of the planner's allocation. The natural question that follows is whether regulation

[19]The finding that the bank takes more leverage than what is socially optimal is not unique to this paper, nor is it our main contribution. Several other studies have related findings, such as Van den Heuvel [2008] and Christiano and Ikeda [2016], for instance. Our goal is to develop the most parsimonious model that has the mechanisms needed to conduct a welfare analysis of stress-test based capital requirements and determine the optimal regulation.

can help improve welfare in the competitive equilibrium.[20] Lemma 3 notes that regulation can achieve the second best outcome.

Lemma 3. *The solution to the constrained planner's problem can be implemented via a minimum capital-ratio requirement. In particular, the regulator must optimally impose a stricter requirement on a bank that, all else equal, exhibits a higher bankruptcy cost.*

Proof. See Appendix B. ■

Intuitively, the reason for this result is that the regulator's decision problem is very similar to that of a constrained planner. While a planner chooses deposits on behalf of the bank to maximise welfare, a regulator imposes a minimum capital-ratio requirement, which implies a given level of deposits when capital is fixed and the requirement is binding.

With this background, we are ready to investigate the choice of optimal regulation in our model. We first characterize the optimal regulation in the full information case, which serves as a benchmark throughout the paper.[21] In this case, the regulator can perfectly observe banks' types on date-1, and can impose perfectly type-contingent capital requirements as follows.

Lemma 4. *The regulator optimally sets a more stringent requirement on the low-type bank as compared to a high-type bank, i.e.* $\chi_L^o > \chi_H^o$.

Proof. See Appendix C. ■

Intuitively, for a given level of deposits, a low-type bank not only generates lower expected output, but is also more likely to fail, which in turn rationalises a stricter regulation for such a bank.

[20] A capital-ratio requirement is not the only regulatory tool that can implement the second best. A tax (or a deposit insurance premium) that is a function of the balance sheet choice of the bank may also achieve the same objective.

[21] Note that even though regulation applies only on date-1, the regulator must announce the requirement on date-0 to enable banks to form expectations on that basis and chooses its date-0 effort accordingly. In this regard, we abstract away from time-inconsistency issues, and assume that regulatory announcements are credible.

We now turn to the study of optimal regulation in the private information case where the bank's type on date-1 is not known to the regulator. In this case the regulator must adopt a uniform capital requirement χ which is applicable irrespective of the bank's type. We begin by establishing a key result of the paper on how banks respond to a change in a binding regulation.[22]

Lemma 5. *Assume that regulation χ binds on date-1 for at least one bank type. Then the effort the bank exerts on date-0 decreases as χ rises.*

Proof. First consider the case where regulation binds for both bank types. As shown in Lemma 1, the bank's date-0 effort e depends on $\omega = V_H(\chi) - V_L(\chi)$, i.e. the wedge between the value of being a high- versus low-type on date-1. The key then to proving this lemma is to characterise how regulation affects ω.

$$\omega = \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f_H(\psi) d\psi - \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f_L(\psi) d\psi$$

where $d = k/\chi$. The derivative of ω with respect to χ gives:

$$\frac{\partial \omega}{\partial \chi} = -\frac{k\beta}{\chi^2} \left(\underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_H(\psi) d\psi}_{\Lambda_H} - \underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_L(\psi) d\psi}_{\Lambda_L} \right) \tag{9}$$

where Λ_s is the Lagrange multiplier on the regulatory constraint in the bank's problem.

To sign this expression, we proceed as follows. First note that since the (binding) regulatory requirement is the same for both types of bank, their deposit choices and thus the failure cutoffs ψ_c are also the same. Then let $\hat{F}_H$ and $\hat{F}_L$ be the distribution functions of ψ for high- and low-type banks, truncated below at ψ_c. Since $\mu_H > \mu_L$ (while the variances

[22]The case where regulation does not bind is not relevant nor interesting because we already showed that an inefficiency rationalises regulation. Moreover, while in practice, banks generally maintain a capital ratio above the *de jure* minimum requirement – to have a management buffer – this can be easily incorporated in our framework by adding a constant to the *de jure* requirement, which would then be the *de facto* requirement.

are the same), $\hat{F}_H$ $FOSD$ $\hat{F}_L$, that is $\hat{F}_H(\psi) \leq \hat{F}_L(\psi)\ \forall\ \psi$. Finally, since $(\psi g'(k+d) - R)$ is an increasing function of ψ, it follows that:

$$\int \Big(\psi g'(k+d) - R\Big) d\hat{F}_H(\psi) - \int \Big(\psi g'(k+d) - R\Big) d\hat{F}_L(\psi) = \Lambda_H - \Lambda_L > 0.$$[23]

In turn, this implies that $\frac{\partial \omega}{\partial \chi} < 0$. Then from Lemma 1 we know that $\frac{\partial e}{\partial \omega} > 0$, which completes the proof since:

$$\frac{\partial e}{\partial \chi} = \frac{\partial e}{\partial \omega} \frac{\partial \omega}{\partial \chi} < 0.$$

In case regulation binds for only one bank – which has to be the high type bank since it chooses a lower capital ratio in the unregulated economy – we reach the same result because V_L does not change with χ while V_H decreases with χ, pushing the wedge ω and the effort lower.

■

The insight offered by Lemma 5 is as follows. Because a high-type bank's assets are more profitable, the opportunity cost of stricter capital requirements is greater for this bank. As such, an increase from a given level of requirement leads to a greater decline in the expected value of the high-type bank than the low type bank. This, in turn, lowers value wedge between low and high type banks, and in turn lowers the return to exerting more effort. Therefore, in contrast to the conventional wisdom that more skin-in-the-game (via higher capital requirement) can induce banks to become safer, we show that under information frictions banks might respond adversely to stricter regulation, and become more risky.

This insight points to an important trade-off the regulator faces while setting χ. A

[23] To prove this formally, consider continuous distribution functions G and H such that $\forall x, H(x) \leq G(x)$, and define $y(x) = H^{-1}(G(x))$. Then for any increasing function $w(x)$, $\int w(y(x)) dH(y(x)) = \int w(y(x)) dG(x)$. Next, note that $y(x) = H^{-1}(G(x)) \implies y(x) \geq x$ since $\forall x, H(x) \leq G(x)$. In turn, since $w(.)$ is an increasing function, $w(y(x)) \geq w(x)$. Thus, $\int w(y(x)) dG(x) \geq \int w(x) dG(x)$. Indeed, intuitively, the *shadow cost* of the minimum capital-ratio constraint should be greater for a bank whose assets are *ceteris paribus* more profitable.

higher χ can improve welfare *ex-post* by reducing the bank-failure inefficiency (reduce default probability).[24] Yet, under imperfect information, a higher χ can reduce welfare due to its adverse impact on the effort the bank exerts *ex-ante*. With this, we are ready to characterise the optimal χ.

Proposition 1. *The optimal requirement χ^o in the case where the regulator cannot observe the bank's type lies between by the optimal full-information (benchmark) requirements for low- and high-type banks: $\chi^o_L \geq \chi^o \geq \chi^o_H$.*

Proof. The problem of a benevolent regulator on date-0 when it cannot impose bank-specific requirements, is as follows:

$$\max_{\chi} \quad \beta p(e) U_H(\chi) + \beta(1 - p(e)) U_L(\chi) - \zeta(e)$$

Here U_s is the household's and banker's combined expected lifetime consumption utilities when the banker turns out to be of type s, while $\zeta(e)$ accounts for the banker's effort on date-0. We will prove the proposition via the method of contradiction. Let χ^o solve the above problem. Then, if $\chi^o > \chi^o_L > \chi^o_H$, it means that the requirement is more strict than the optimal requirement for both bank types, and thus a lower χ^o would improve welfare in case of each bank type, as well as the total expected welfare. Similarly, if $\chi^o_L > \chi^o_H > \chi^o$, it means that the requirement is more liberal than the optimal requirement for both bank types, and thus a higher χ^o would improve total welfare. ■

Intuitively, this proposition shows that when there is information asymmetry, the regulator chooses a *middle-ground* relative to the optimal bank-specific requirements in the full information case.

[24] This is especially in case of a low-type bank where the welfare reducing effect of a higher χ via lower expected output is dominated by its welfare improving effect via lower default risk).

4 Optimal Regulation with Supervision

We now introduce stress-tests as a supervisory tool that helps (partially) overcome information frictions. In turn, this allows the regulator to better align the capital requirement to banks' types. We first analyse the optimal capital surcharge based on stress-tests of given degree of accuracy, and then study how stress-test accuracy can be optimized.

4.1 Optimal surcharge

We are concerned with the welfare maximising level of surcharge x on top of χ^o (for banks that fail in the test) that the regulator must announce on date-0. Assuming that the stress-test is not perfect (that is low-type banks can pass and high-type banks can fail the test), the choice of x is subject to a three-way trade-off.

1. In case of the low-type bank, the surcharge (upon failing the test) *increases* welfare as long as $x \leq \chi_L^o - \chi^o$. This is because the surcharge brings the effective requirement $(\chi^o + x)$ closer to the optimal for a low type bank (χ_L^o).

2. In case of the high-type bank, the surcharge (upon failing the test) *decreases* welfare. This is because $\chi^o + x > \chi^o \geq \chi_H^o$, as a result of which the surcharge takes the effective requirement away from the optimal for a high-type bank (χ_H^o).

3. The surcharge affects the wedge between the expected value of being high- versus low-type on date-1, and thus impacts the bank's behaviour on date-0. Depending on the accuracy of the stress test, this can lead to an increase or decrease in the bank's effort, as we prove in Lemma 6 below. Accordingly, a higher surcharge can *increase or decrease* welfare through its effect on effort.

Lemma 6. *The bank's effort may increase or decrease with a surcharge, depending on the accuracy of the stress test.*

Proof. The date-0 problem of the bank is:

$$\max_{e} \quad -\zeta(e) + \beta p(e) \underbrace{\left(q_H V_H(\mathcal{X}^o) + (1-q_H) V_H(\mathcal{X}^o + x)\right)}_{\mathbb{E}V_H} + \\ \beta(1-p(e)) \underbrace{\left(q_L V_L(\mathcal{X}^o) + (1-q_L) V_L(\mathcal{X}^o + x)\right)}_{\mathbb{E}V_L} \tag{10}$$

We begin by noting that similar to the case without stress testing, the effort the bank exerts increases with the *expected* value function wedge $\omega = \mathbb{E}V_H - \mathbb{E}V_L$. Taking the derivative of ω with respect to x at $x = 0$ gives:

$$\left.\frac{\partial \omega}{\partial x}\right|_{x=0} = (1-q_H) V_H'(\mathcal{X}^o) - (1-q_L) V_L'(\mathcal{X}^o)$$

where V' indicates the derivative of the value function. To determine the sign of this expression, divide everything by $V_L'(\mathcal{X}^o)$:[25]

$$sgn\left(\left.\frac{\partial \omega}{\partial x}\right|_{x=0}\right) = -sgn\left((1-q_H)\underbrace{\frac{V_H'(\mathcal{X}^o)}{V_L'(\mathcal{X}^o)}}_{\nu} - (1-q_L)\right)$$

Next, recall from the proof of Lemma (5) that $V_H'(\mathcal{X}^o) - V_L'(\mathcal{X}^o) < 0$, which implies that $\nu > 1$ since $V_H'(\mathcal{X}^o) < 0$ and $V_L'(\mathcal{X}^o) < 0$. Thus, the effect of surcharge on the bank's effort choice depends on the accuracy of the test as follows:

$$(1-q_L) - (1-q_H)\nu \begin{cases} > 0 & \Longrightarrow \quad \text{efforts increases with surcharge} \\ = 0 & \Longrightarrow \quad \text{efforts does not change with surcharge} \\ < 0 & \Longrightarrow \quad \text{effort decreases with surcharge} \end{cases}$$

■

[25]Since the value of a more regulated bank is lower, $V_L'(\mathcal{X}^o) < 0$. As such, we add a minus sign to the RHS expression.

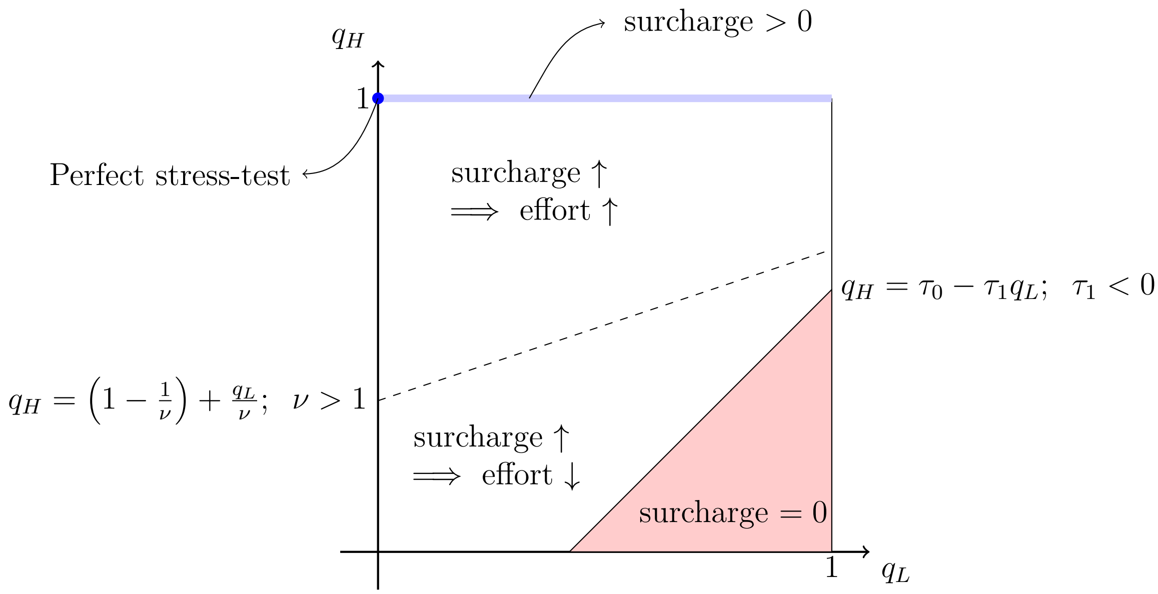

Figure 2: **Stress-test accuracy, effect on ex-ante effort, and optimal penalties:** Each point on the unit square characterises the accuracy of a stress-test. A higher q_H and a lower q_L indicate a more accurate test. The optimal surcharge is zero if accuracy is below the solid diagonal line (i.e. in the shaded area), and positive if $q_H = 1$. Effort increases with the surcharge above the dotted diagonal line, and decreases otherwise.

Intuitively, ν captures the relative shadow cost of tightening regulation for the high- and low-type banks. *Ceteris paribus*, a higher ν makes imposing a surcharge less desirable by making it more likely that the bank reduces effort. Similarly, for a given ν, a higher Type-I (i.e. lower q_H) or Type-II error rate (higher q_L) would make $(1-q_L)-(1-q_H)\nu$ more negative and cause the bank to reduce effort following a higher surcharge. Indeed, if a high-type bank is sufficiently likely to fail the stress-test and the low-type bank is sufficiently likely to pass, then the high-type bank will often face a surcharge while the low-type bank will not, thereby reducing the relative benefit to being a high-type bank. This will induce the bank to exert less effort towards becoming high-type in the first place. Relatedly, it is clear from Lemma 6 that with a perfect stress test, i.e. when $(q_H = 1, q_L = 0)$, effort increases with surcharge, while when $q_H = q_L = 0.5$, effort decreases with surcharge. We indicate these qualitative insights in Figure 2.

We are now ready to assess the relationship between a given level of stress-test accuracy and the optimal surcharge. We begin by noting the following result.

Proposition 2. *No surcharge must be imposed if a linear combination of the Type-1 and Type-II error rates is higher than a cutoff.*

Proof. See Appendix D. ■

Intuitively, the proposition shows that when q_H is low and/or q_L is high – both of which reflect a relatively less accurate stress-test – the surcharge must be zero. This case in indicated by the shaded area in Figure 2. Finally we explore conditions under which the optimal surcharge can be positive.

Consider a stress-test that is accurate in identifying high-type banks i.e. $q_H = 1$, but is possibly inaccurate in identifying low-type banks i.e. $1 > q_L \geq 0$. In this case a higher x does not affect $\mathbb{E}V_H$, but decreases $\mathbb{E}V_L$ (recall equation (10)). As a result, the banker increases effort as surcharge increases. Then we consider the regulator's problem:

$$\max_{x} \quad \beta p(e) U_H(\chi^o) + \beta(1 - p(e)) \Big(q_L U_L(\chi^o) + (1 - q_L) U_L(\chi^o + x) \Big) - \zeta(e)$$

A higher x does not affect welfare when the bank passes the test, but increases welfare when it fails the test as long as $x \leq \chi^o_L - \chi^o$ (recall from Proposition 4 that beyond this threshold, the effective requirement on the low-type bank is higher than the optimal requirement χ^o_L.)

Combining the effect of a surcharge on effort e and $U_L(\chi^o + x)$, both of which increase as x increases, and given that $U_H(\chi^o) > U_L(\chi^o)$, it is clear that welfare, ignoring the effect of the surcharge on the cost of effort, must increase as x rises above zero. Thus, if the cost of effort is sufficiently small, the optimal surcharge must be strictly positive. Together with proposition 2, this insight points to a material shift in the relation between optimal surcharge and stress-test accuracy, with the optimal surcharge being zero (positive) if the level of accuracy of the stress tests is sufficiently low (high).

In what follows, we formalise this insight using a simpler version of the model where the probability that a bank is of a given type is fixed.[26]

Proposition 3. *The optimal surcharge increases with stress-test accuracy.*

Proof. The regulator's problem in this case is as follows:

$$\max_{x} \quad \beta p\Big(q_H U_H(\mathcal{X}^o) + (1-q_H)U_H(\mathcal{X}^o + x)\Big) + \beta(1-p)\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big)$$

The first order condition is:

$$[x] \quad 0 = p(1-q_H)U_H'(\mathcal{X}^o + x) + (1-p)(1-q_L)U_L'(\mathcal{X}^o + x)$$

Next consider an increase in accuracy via a higher q_H (the proof in case of a lower q_L is similar):

$$0 = -pU_H'(\mathcal{X}^o + x) + p(1-q_H)U_H''(\mathcal{X}^o + x)\frac{\partial x}{\partial q_H} + (1-p)(1-q_L)U_L''(\mathcal{X}^o + x)\frac{\partial x}{\partial q_H}$$

Since U is concave, and $U_H'(\mathcal{X}^o + x)$ is negative (because $\mathcal{X}^o$ is higher than the optimal requirement for the high-type bank), $\frac{\partial x}{\partial q_H} > 0$. ■

Next, we examine how the optimal surcharge changes as bankruptcy cost increases. Bankruptcy costs exacerbate the trade-off for regulators. A higher surcharge may be justified on the grounds that it lowers the failure rate and thus the expected bankruptcy cost. Yet, a stress test that is not sufficiently accurate may render a higher surcharge sub-optimal. While it is not possible to generally characterise the optimal shift in surcharge as bankruptcy costs increase, we assume that the probability of being of a given type is fixed, and note the following result.[27]

[26] We cannot prove this result analytically in the fully specified model, even though numerical simulations show that the result continues to hold generally.

[27] We study quantitatively in Section 5 how the surcharge responds to an increase in bankruptcy costs in the fully specified model.

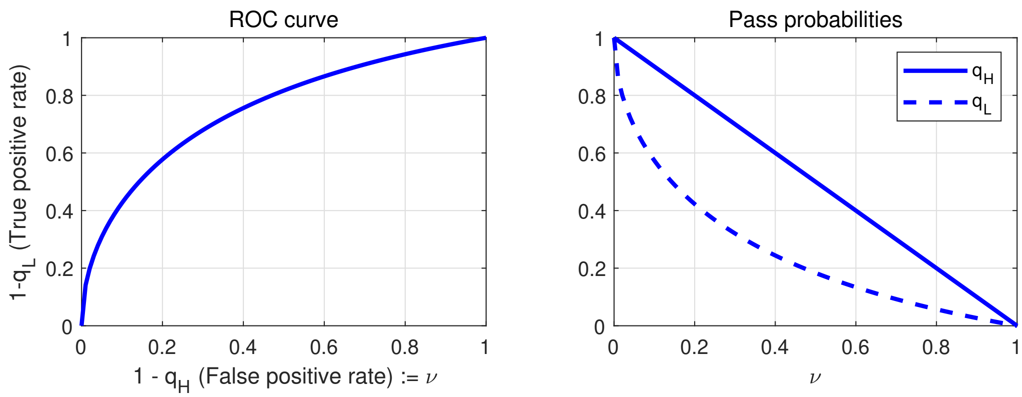

Figure 3: The mapping from a given ROC curve to pass probabilities for high- and low-type banks.

Proposition 4. *The optimal surcharge increases as bankruptcy cost increases.*

Proof. See Appendix E. ■

To conclude this subsection, we find that when stress-test accuracy is low, capital requirements should optimally not be based on stress-test results – a finding that is consistent with the literature. Yet, when stress-test accuracy is high, we show – in contrast to Ahnert et al. [2020] and Morrison and White [2005] for example – that capital surcharges should be higher. The difference in our conclusions stems from the distinct roles of regulation and supervision: they are substitutes in Ahnert et al. [2020] and Morrison and White [2005] while they are complements in this paper. Indeed, capital surcharges in this paper are not only a means to discipline banks' ex-ante behaviors, but they are also serve as ex-post corrective devices. This is key because in our paper riskier banks are not eliminated from the market, but continue to exist in the equilibrium.

4.2 Optimal stress-test accuracy

Thus far, we have considered the accuracy of the stress test – as summarised by (q_H, q_L) – to be given exogenously. In reality, supervisors design the various aspects of stress-tests and thus may be able to choose q_H and q_L, although not necessarily independently. Indeed, policymakers have remarked on the need to continually optimise stress-test design.[28] Relatedly, the academic literature has noted constraints and trade-offs in stress-test design [Parlatore and Philippon, 2020; Leitner and Williams, 2022]. For example, if stress scenarios are too extreme or unlikely, they may not be a good gauge of banks' resilience. An average scenario is not ideal either as it may not be informative about banks' health in tough conditions and may lead to faulty assessment. Likewise, the degree to which the statistical model that translates the stress-scenario into projections of banks' capital ratios exhibits amplification dynamics matters for the Type-I and II error rates. In the end, the hypothetical scenario and the projection models (of the bank and the regulator) jointly determine the Receiver Operative Characteristics (ROC) curve of the stress-test, ie the set of mutually consistent q_H and q_L (or equivalently, the mutually consistent Type-I and II error rates). This means that policmakers face the following related questions when designing the test:

1. Where on a given ROC curve should the supervisor aim to position itself – i.e. which (q_H, q_L) tuple on the ROC curve should it choose. And what capital surcharge should the regulator impose on failing banks in this case.

2. More generally, which ROC curve should the supervisor choose given the potential technical, logistic, or fundamental constraints in improving the accuracy of a stress-test [Eisenbach et al., 2016].

Case I: Optimal position on a given ROC curve Moving along the ROC curve entails reducing one error rate (say Type-I or false positive rate) while increasing the other (Type-II or false negative rate), as illustrated in Figure 3. Denoting via ν the position along the ROC curve – which in turn is related to the average difficulty in passing the test – the figure shows that increasing ν to reduce q_L (the probability that a low-type bank

[28] For example, Mr Jerome H Powell, Chair of US Federal Reserve System said " ... the tests will need to vary from year to year ... if the stress tests do not evolve, they risk becoming a compliance exercise ... " at the research conference titled "Stress Testing: A Discussion and Review" on 9 July 2019.

passes the stress-test) comes at the cost of lowering q_H (the probability that a high-type bank passes the stress-test). The optimal combination of q_L and q_H then depends on the welfare implications of the associated error rates. For example, not identifying a low-type bank (Type-II error) and thus not imposing a capital surcharge means that the low-type bank would remain highly leveraged and continue to pose the bank-failure inefficiency. Likewise, deeming a high-type bank as low-type (Type-I error) and imposing a higher capital surcharge would inefficiently constrain an otherwise healthy bank.

To characterise the optimal supervisory stringency (i.e. choosing the optimal ν) for a given ROC curve, we consider the following joint problem of a regulator and a supervisor who choose respectively x and ν. Note that we continue to assume that the probability of a given type of bank is fixed:

$$\max_{x,\nu} \quad \beta p\Big(q_H(\nu)U_H(\mathcal{X}^o) + (1 - q_H(\nu))U_H(\mathcal{X}^o + x)\Big) +$$
$$\beta(1-p)\Big(q_L(\nu)U_L(\mathcal{X}^o) + (1 - q_L(\nu))U_L(\mathcal{X}^o + x)\Big)$$

The first order condition (FOC) w.r.t. ν is:

$$[\nu] \quad 0 = \beta p q_H'(\nu)(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) + \beta(1-p)q_L'(\nu)(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x))$$

$$\implies \underbrace{q_L'(\nu)}_{<0} = \frac{p\,\overbrace{(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x))}^{>0}}{(1-p)\,\underbrace{(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x))}_{<0}} \tag{11}$$

where we use the fact that $q_H(\nu) = 1 - \nu$. Note also that $q_L(\nu)$ is a decreasing and convex function. We are interested in how the optimal choice of ν is affected by a change in a model parameter, say ρ, which could be the standard deviation of return on assets, σ, or the cost of bankruptcy, Δ, both of which are directly related to the *expected* loss of bankruptcy. To this end, we take the total derivative of the FOC w.r.t. ρ and note how ν

responds to ρ in Proposition 5:

$$[\nu]: \quad \underbrace{q_L''(\nu)}_{>0} \frac{d\nu}{d\rho} = \frac{d}{d\rho} \frac{p(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x))}{(1-p)(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x))}$$

Proposition 5. *In response to an increase in the expected loss of bank default (loss given default or probability of default), assuming the surcharge remains fixed, the supervisor chooses a smaller false negative rate* (q_L)*, even though this means that the false positive rate* $(1 - q_H)$ *is higher.*

Proof. See Appendix F. ■

Intuitively, an increase in the expected cost of bankruptcy makes not imposing a surcharge on a low-type bank (which has a higher probability of failing *ceteris paribus*) socially more costly. As a result, the supervisor optimally errs on the side of caution. That is, it makes the stress-test generally more difficult to pass, which means that more high-type banks also end up failing the test, but the welfare benefit of fewer low-type banks remaining undetected and without a surcharge dominates.

While it is not possible to analytically characterise the jointly optimal response in terms of q_H, q_L and x, we know from Proposition 4 that the optimal surcharge is higher when bankruptcy cost is higher. Numerical simulations in Section 5 suggest that the findings in Propositions 5 and 4 are likely to continue to hold in the joint optimisation.

Case II: Choosing the ROC curve Pushing out the ROC curve towards the northwest corner in Figure 4 (or choosing the shape of the ROC curve more generally) could allow the supervisor to reduce both error rates at the same time. But designing such a test is likely to be subject to costs and constraints. For one, a more extensive review of the banks' balance sheet and its risk models would not only absorb additional supervisory force, but also more bank resources. In addition, there may be fundamental constraints in accurately predicting how a bank would perform under stress or the risks it poses to

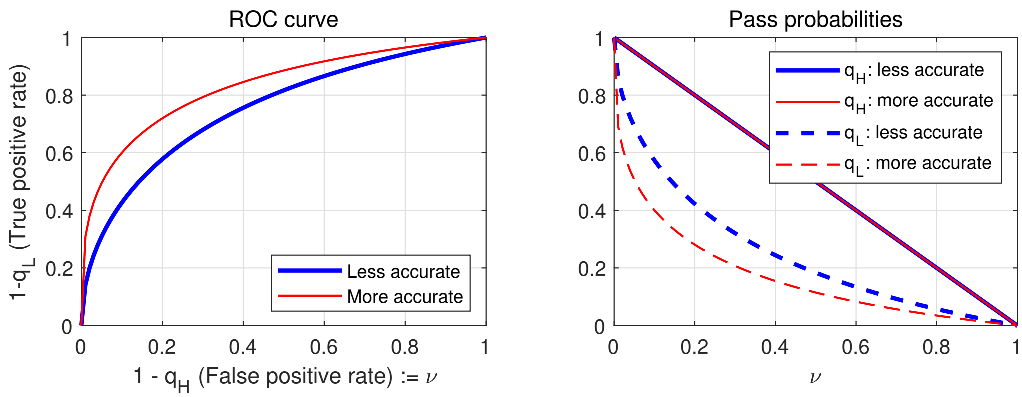

Figure 4: The mapping from a given ROC curve to pass probabilities for high- and low-type banks.

the wider economy, not least because such an assessment would inherently depend on a number of assumptions.

To model these trade-offs, we consider the joint problem of the regulator and the supervisor where they choose a test-design parameter θ that governs the area under the ROC curve (AUROC)[29], the position parameter ν that pins down the choice of q_H, q_L along that curve, and the optimal surcharge x. In line with the discussion above, we assume that improving the AUROC entails a social cost $\mathcal{C}(\theta)$ where $\mathcal{C}$ is some increasing function, and note the following result.

Proposition 6. *As the cost of improving accuracy decreases, the supervisor increases the average rate of passing for both types of banks, ie chooses a lower q_H (higher false positive rate) and a lower q_L (lower false negative rate). The optimal response for the regulator in terms of the surcharge x is ambiguous, and depends on the marginal impact on welfare of a surcharge imposed (correctly) on low- and (incorrectly) on high-type banks.*

Proof. See Appendix G. ■

Intuitively, while a lower cost of accuracy enables the supervisor to make passing the test easier on average, the impact on surcharge is ambiguous because while the false positive

[29] See Hanley and McNeil [1982] for discussion on why the area under the ROC is a useful statistic to assess the overall accuracy of a binary classifier.

rate declines, the false negative rate increases. Nonetheless, we assess the direction of change in surcharge in a calibrated version of the model in Section 5.

Overall, the findings in this subsection underscore that the various elements of banking oversight, i.e. supervision and the attendant capital regulation, are intricately linked and should be optimised jointly due to the potential for adverse incentives otherwise.

4.3 Trade-offs in disclosing stress-test result

A contrasting aspect of stress-testing compared to other forms of micro-prudential supervision and regulation is that the testing methodology and test results are disclosed to the wider public in quite some detail. Disclosure of results can have an additional impact on banks via market discipline i.e. the difference between how investors perceive a bank relative to its stress-test performance. Depending on the direction in which investors update their priors, they may seek a higher or lower return when providing funding to banks. This can impact how banks respond to stress-tests, and have implications not only for stress-test disclosure policy, as discussed in [Goldstein and Sapra, 2014; Goldstein and Leitner, 2018], but also for how test-based capital requirements must be set.[30]

To assess this latter aspect, we extend our model to include a role for uninsured investors that react to stress-test results. To create an incentive for the bank to pursue the two types of funding, we assume that deposit based funding is not easily scalable, and thus the unit cost of deposit funding $R(d)$ increases with the funding amount. At the same time, investor funding w, even though more costly for smaller amounts, is easily scalable, and is the relatively cheaper source of financing for larger amounts (see Figure 5). Yet, when a bank fails the stress-test, while insured depositors do not seek a higher return, uninsured investors raise their required return $Q(w)$ by, say, δ.[31] The date-1 problem of the bank in

[30] Other studies in this literature include Corona et al. [2019] who assess how bailout regime and disclosure policy interact, Orlov et al. [2018] who characterise the optimal disclosure policy for high- and low-risk banks, and Bouvard et al. [2015] who show that the optimal disclosure policy must vary along the business cycle.

[31] Relatedly, Chen et al. [2020] provide empirical evidence of the fact that uninsured deposit flows are

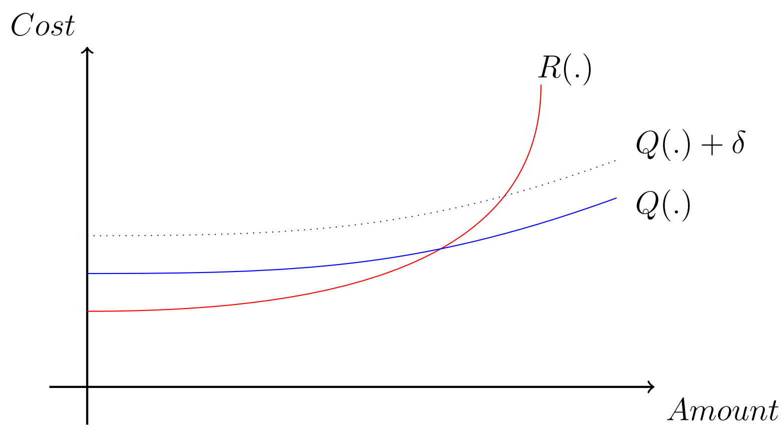

Figure 5: Cost of insured deposits and uninsured investor funding

this case is as follows:

$$V_s(\chi_s) = \max_{d,w} \quad \beta \int_{\frac{R(d)d+Q(w)w}{g(k+d+w)}}^{\infty} \Big(\psi g(k+d+w) - R(d)d - Q(w)w\Big) f_s(\psi) d\psi$$
$$s.t. \quad \frac{k}{\chi_s} \geq (d+w). \tag{12}$$

Assuming that both forms of financing are used in equilibrium, we assess the implications for banks and for the regulator. We first note that failure in the test is now more costly for the bank – not only does it need to satisfy a higher capital ratio, its unit cost of funding is higher compared to the case where disclosures have no material impact (i.e. $\delta = 0$). Formally, the FOCs of the bank's problem imply that d and w are determined in the case of passing and failing banks as follows, respectively:

$$\frac{k}{\chi} = d + w; \quad R'(d)d + R(d) = Q'(w)w + Q(w)$$

$$\frac{k}{\chi + x} = d + w; \quad R'(d)d + R(d) = Q'(w)w + Q(w) + \delta$$

To make analytical progress, we assume simple forms of the cost functions: $R(d) = R_0 + R_1 d$ and $Q(w) = Q$ such that they continue to reflect the underlying intuition that investor

more sensitive to information about bank performance.

funding is more elastic than deposit funding. Solving the FOCs explicitly leads to:

$$d_{pass} = \frac{Q - R_0}{2R_1}; \quad w_{pass} = \frac{k}{\chi} - \frac{Q - R_0}{2R_1};$$

$$d_{fail} = \frac{Q + \delta - R_0}{2R_1}; \quad w_{fail} = \frac{k}{\chi + x} - \frac{Q + \delta - R_0}{2R_1};$$

That is, upon failure in the test, the bank reduces its overall balance sheet and funding, and tilts its funding composition towards deposits. At the same time, the total funding cost (TC) of a failing bank is increasing in δ.[32] In turn, the value of a failing bank is decreasing in δ.

To derive the implications for the effort the bank exerts on date-0, we assess the impact of δ on the expected value function wedge (recall equation 6). The value of a high- or low-type bank that passes the test – ie $V_H(\chi^o)$ and $V_L(\chi^o)$ – remains unaffected by δ. However, δ leads to a larger decline in the value of a high-type bank that fails the test. To see this formally, consider the resolved value function of the s-type bank, where we have already solved for the d, w decisions as a function of δ:

$$V_s(\chi^o + x; \delta) = \beta \int_{\frac{TC(\delta)}{g(k+d+w)}}^{\infty} \Big(\psi g(k + d + w) - TC(\delta)\Big) f_s(\psi) d\psi$$

The derivative of the value function with respect to δ implies:[33]

$$\frac{dV_s(\chi^o + x; \delta)}{d\delta} = -\beta TC'(\delta) \int_{\frac{TC(\delta)}{g(k+d+w)}}^{\infty} f_s(\psi) d\psi$$

The above expression proves that the value function of each type of bank is decreasing in δ since $TC'(\delta) > 0$. Moreover, since the failure cutoff and $TC(.)$ are independent of

[32]To see this, consider the total funding cost (TC) of a failing bank as a function of δ : $TC(\delta) = R(d)d + (Q + \delta)(k/(\chi + x) - d)$ where $d = (Q + \delta - R_0)/(2R_1)$. Taking the derivative of the above expression with respect to δ immediately leads to the above result: $TC'(\delta) > 0$.

[33]Note that $d + w$ is independent of δ and that only one of the terms following an application of the Leibniz rule is non-zero.

Parameter	Description	Value	Target moments	Value
α	Payoff exponent: $(\text{k+d})^{\alpha}$	0.9668	Gross return on assets	5.84%
μ	Mean of ψ	1.1501	Equity capital to assets ratio	8.17%
σ	Standard-deviation of ψ	0.0549	Value-at-risk threshold	1%
$\bar{Y}$	Household income	43.701	Household savings rate	25.72%
β	Discount factor	0.9760	Deposit interest-rate	2.46%
Δ	Failure cost	0.22	US bank failure losses	22%

Table 1: Parameter values and target moments. The mean gross return on assets and capital ratio are computed using data on the top 3 or more banks in each G20 country, and is sourced from Fitch. The household savings rate is calculated as the average domestic savings to GDP ratio from the World Bank. Deposit rate is also sourced the World Bank, and data on bank failure losses are from the FDIC. Note that the last two parameters and target moments have a one-to-one mapping (i.e. they need not be estimated jointly), and that without loss of generality k is normalised to unity. The value of the moments in data exactly match those implied by the model.

bank type, the decline in value is greater in case of the high-type bank. Intuitively, the probability that the high-type bank is solvent is higher, which means that it is more likely to incur the higher funding cost. Therefore, it follows that $V_H(\mathcal{X}^o+x;\delta=0)-V_H(\mathcal{X}^o+x;\delta>0) > V_L(\mathcal{X}^o+x;\delta=0)-V_L(\mathcal{X}^o+x;\delta>0)$. As such, *ceteris paribus,* a higher δ depresses the expected value function wedge.

The above analysis shows that depending on the accuracy of stress-tests, disclosing the results of the test can strengthen or worsen the bank's ex-ante incentives. When the test is sufficiently accurate, the disclosure can help improve market discipline and increase the effort banks' exert ex-ante. Yet, when the test is less accurate (ie moving south-east from the north-west corner in Figure 2), disclosure can worsen ex-ante incentives, and place further constraints on using stress-test results for imposing bank-specific capital surcharges.

5 Numerical illustration

We now calibrate the model parameters. Our goal is not to draw empirical predictions, but to provide a relevant numerical illustration of our analytical results. To this end, we set the parameters such that model generated moments are equal to the corresponding data moments (see Table 1). As an example, we use data on G20 member countries. Unless otherwise mentioned, for each data moment, we consider the average across these

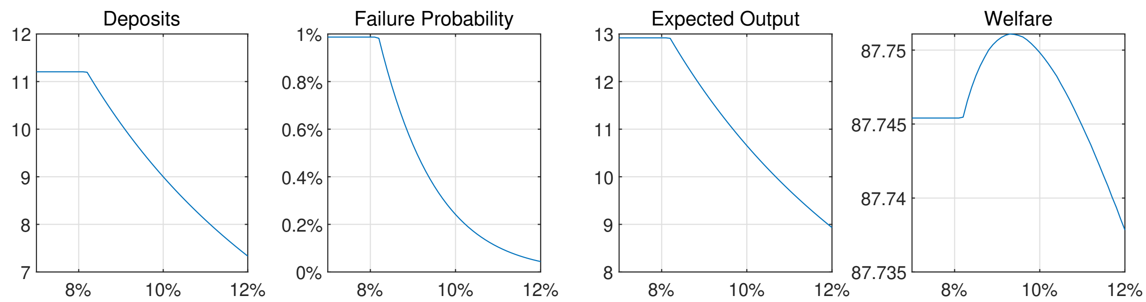

Figure 6: The effect of minimum capital-ratio requirement (x-axis) on the high-type bank and on overall welfare.

countries. We focus on data from 2019 to abstract away from any Covid-19 crisis related aberrations in the data. We consider the following moments as targets. First is the gross return on assets, which takes into account interest as well as non-interest income. Second is the equity capital to assets ratio. Third is a typical regulatory or bank-management-imposed value-at-risk threshold of 1%. Together, these three moments capture aspects of a bank's financials that are key for their response to regulation. The fourth moment is the household savings rate, approximated as the domestic savings to GDP ratio. Fifth is the deposit interest rate. Finally, Δ is set in line with the losses associated with bank failures in the US in the period after the Great Financial Crisis.[34] According to the Federal Deposit Insurance Commission (FDIC), there have been 367 bank failures during this period, and the median estimated loss is about 21% of the failed bank's assets, while the inter-quartile range is 13% to 30%. Our target moment is the mean, which is 22%.

As regards the functional forms, we assume the cost of exerting effort by the bank on date-0 as $\zeta(e) = \gamma_e e^2$, and the related probability of the bank becoming a high-type on date-1 as $p(e) = 1 - 1/(1+e)$. The exact functional forms do not matter for our qualitative results as long as $\zeta(.)$ is (weakly) convex and $p(.)$ is concave. We choose γ_e so that the bank is high- or low-type with equal probability. As regards μ_H and μ_L, we assume a symmetric perturbation of 25 basis points around μ. Finally, we treat q_H and q_L as free parameters

[34] Data on bank failure related losses is not available systematically for all the countries in our sample; hence we only use the US data for this moment.

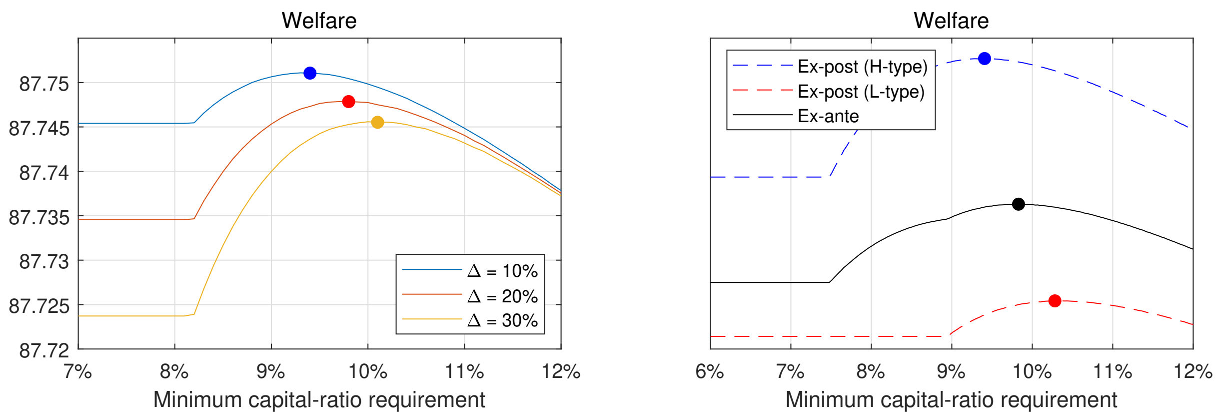

Figure 7: *Left-hand panel*: The welfare maximizing regulation for varying levels of bank failure costs. *Right-hand panel*: Optimal ex-post requirement depending on bank type, and the optimal ex-ante requirement in the absence of stress tests.

that we conduct comparative statics with respect to.

Optimal regulation under full information We begin by analyzing the impact of a minimum capital-ratio requirement on the bank's behavior and overall welfare on date-1. Without loss of generality, we focus on a high-type bank. Starting from the unregulated economy, a higher minimum capital-ratio requirement forces the bank to deleverage (first panel in Figure 6). This reduces the failure probability (second panel), but also lowers expected output (third panel). The overall effect – one that weighs welfare gains from lower bank failure against the welfare loss from lower expected output – is an inverted U-shaped welfare profile as a function of χ. This finding is consistent with Lemma 3 where we showed that the unregulated equilibrium is sub-optimal and that a minimum capital-ratio requirement can improve welfare, and also with the broader literature (e.g. Begenau [2020], Christiano and Ikeda [2016]).[35] Relatedly, as bank failure costs increase, not only is the optimal requirement higher (as proven in Lemma 3), the welfare gain from regulation is also higher (see left-hand panel in Figure 7).

Finally, we compare the optimal requirement for low- and high-type banks. Consistent

[35]Given the stylized nature of our model, the simulations are not meant to pin down the optimal level of capital requirements in the real world, but rather to illustrate the comparative statics of the model with respect to calibrated parameters.

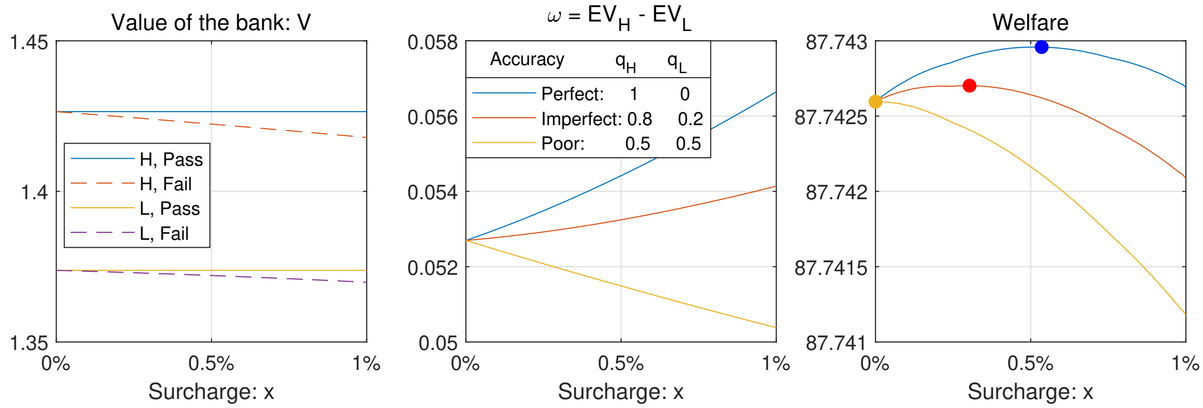

Figure 8: *Left-hand panel:* The value V of the bank in various cases as a function of the surcharge. *Centre panel:* The expected value wedge changes in response to the surcharge for different levels of accuracy of the stress test. *Right-hand panel:* Optimal surcharge.

with Lemma 4, we find that the requirement is higher for the low-type bank (see right-hand panel of Figure 7, dotted lines).

Optimal regulation under information frictions When the regulator cannot observe banks' types ex-post, the optimal ex-ante requirement announced on date-0 cannot be bank-type specific. Consistent with Proposition 1, we find that it is saddled by the ex-post optimal requirements (see solid line in the right-hand panel of Figure 7). Next we assess how a stress-test led surcharge affects bank's behavior. A higher surcharge decreases the value of both high- and low-type banks (left-hand panel of Figure 8). The decrease is starker for a high-type bank – indeed the opportunity cost of not being able to use its balance sheet capacity is higher for a bank whose assets have a higher return. And as long as the stress test is not fully perfect, both $\mathbb{E}V_H$ and $\mathbb{E}V_L$ decrease as x increases.

The difference between $\mathbb{E}V_H$ and $\mathbb{E}V_L$, namely ω – as we showed in Lemma 6 – can increase or decrease depending on the accuracy of the test (see centre panel of Figure 8). This immediately means that the effort banks exert can also increase or decrease as the surcharge is raised (recall that e depends on ω; see the proof of Lemma 5). This is a key insight of the paper – a higher surcharge may not necessarily act as a disciplining device if the basis on which the surcharge is imposed is not sufficiently accurate.

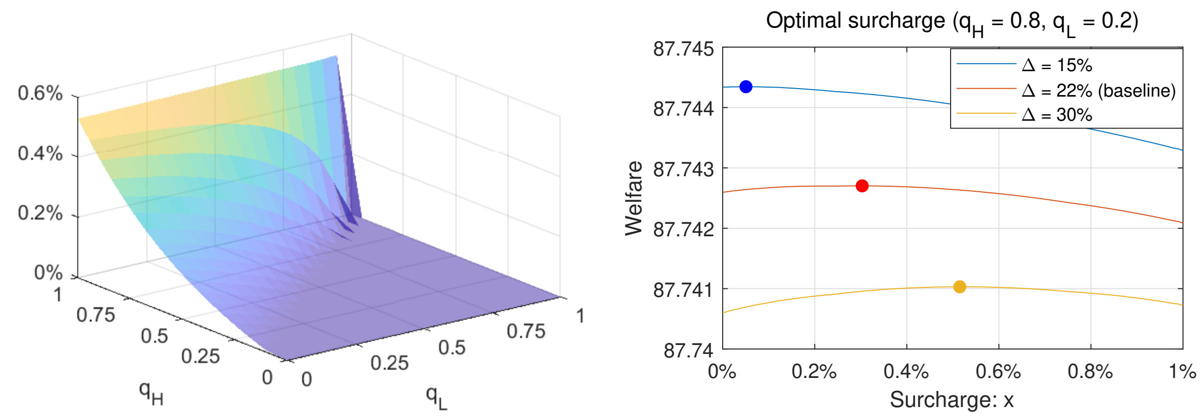

Figure 9: *Left-hand panel*: Optimal surcharge as a function of the accuracy of the stress-test. *Right-hand panel*: Change in optimal surcharge as the cost of failure increases.

Optimal surcharge For an uninformative or less informative stress-test, consistent with proposition 2, the optimal surcharge is zero (right-hand panel of Figure 8). Beyond that, the optimal surcharge increases as stress-test accuracy increases (recall Proposition 3). Overall, the optimal surcharge depends on the following trade-off. Higher capital charges for banks that fail the assessment improve welfare to the extent that low-type banks are penalised, but it reduces welfare by asking some high-type banks to raise unnecessary capital. As such, when the test is sufficiently noisy, capital surcharges may not increase expected welfare. In particular, they may induce banks to reduce their effort.

We compute the optimal surcharge for each accuracy level of the stress-test in the left-hand panel of Figure 9, thus confirming the broad indications sketched in Figure 2. In particular, a phase shift is evident: for sufficiently low levels of accuracy, the optimal surcharge is zero. Moving closer to a perfect stress test ($q_H = 1, q_L = 0$) increases the size of the optimal surcharge. Relatedly, in line with Proposition 4, we show that as the failure cost increases the optimal surcharge also increases (right-hand panel of Figure 9).

Optimal supervisory accuracy Finally we illustrate the case where the supervisor can choose the accuracy of the stress test (recall discussion in subsection 4.2). We assume that the ROC curve is given as $ROC(\nu, \theta) = 2(1 - 1/(1 + \nu))^{1/\theta}$ where $\nu \in [0, 1]$ is the

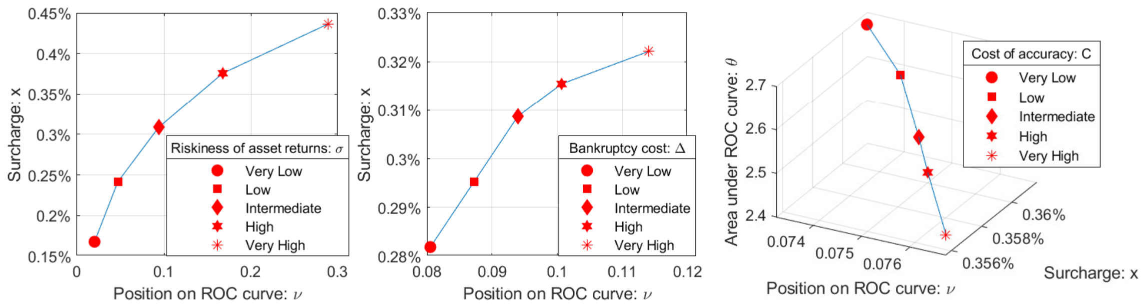

Figure 10: **Optimal stress-testing choices:** In each panel, we assess the comparative statics with respect to a parameter of interest for five different levels of that parameter around its baseline value. For example, in case of σ, we consider values ranging from 0.95 to 1.05 of its baseline value.

position parameter and $\theta \in [1, \infty]$ is a parameter that determines the area under the ROC curve (AUROC).[36] We first assess what a change in the expected cost of bank default means for the optimal position on the ROC curve and the attendant surcharge. Consistent with Proposition 5, we find that as the probability or cost of bank default increases, the regulator makes the test generally more difficult to pass ie choose a higher ν (see the first two panels in Figure 10). This lowers the false negative rate, i.e. reduces the chances that a low-type bank remains undetected and continues to operate at a low level of capital – which is more detrimental for social welfare when the cost of default is higher. While this means worsening the false positive rate, which has a welfare reducing effect as some high-type banks that fail the stress-test would have to hold inefficiently high levels of capital, the former effect dominates. Moreover, in each case, the regulator optimally increases the surcharge which helps reduce banks' probability of default and makes them safer.

Next we consider the case where the regulator can incur a cost $\mathcal{C}(\theta) = C\theta$ to improve the AUROC. We are interested in learning how the supervisor and regulator would adjust the three main elements of their joint oversight, ie AUROC, the position on the ROC curve, and the surcharge, as the cost of accuracy changes (see the right-hand panel in Figure 10). In line with Proposition 6, we find that as C increases, the regulator adopts a lower AUROC (lower θ, z-axis) ie a less accurate stress-test, and makes the test more difficult

[36] The function chosen to denote the ROC curve is meant to capture a typical ROC curve and is not meant to be general.

to pass (higher ν, y-axis). Furthermore, it imposes a smaller surcharge (lower x, x-axis). Intuitively, when the stress-test is less accurate, the risk that banks are mis-classified is higher, in which case a higher surcharge can have a more severe adverse impact on the effort banks exert ex-ante. In turn, this rationalises a smaller surcharge.

6 Covid-19 crisis: A test of risk-assessment tools?

In this section, we use the supervisory stress-tests in the U.S. as an illustration of the noise inherent in risk-assessment exercises. In particular, we compare the findings of the stress-test conducted in 2020, right before the Covid-19 crisis, with the actual impact of the crisis on banks'.[37]

Stress-tests evaluate whether banks have sufficient capital to absorb losses resulting from adverse economic conditions.[38] In the US, the Federal Reserve imposes a capital surcharge on banks based on their performance in the test, as measured by the projected decline in their Common Equity Tier 1 (CET1) capital ratios in the severely adverse scenario. Banks that perform poorly face a higher Stressed Capital Buffer (SCB).

The hypothetical scenario in the 2020 stress-test in the US comprised of a peak unemployment rate of 10 percent, a decline in real GDP of 8.5 percent, and a drop in equity prices of 50 percent through the end of 2020, among other macroeconomic developments.[39] Thirty-three entities participated in the test and the results were published in June 2020. The average decline in CET1 ratio was 2.7 percentage points. Figure 11 shows that while all assessed banks faced a minimum SCB of 2.5%, there is a strong link between stress-test

[37] We acknowledge that the comparison is somewhat "unfair" because stress-tests are not designed to predict crises, and only test banks' capital adequacy in a single hypothetical stress scenario. Yet, the exercise is useful to shed some light on whether there is concordance in banks' riskiness in stress-tests and in the crisis.

[38] This involves projecting revenues, expenses, losses, and, crucially, the capital ratios of the participating banks in a hypothetical recession. The projections use a standard set of capital action assumptions.

[39] The severely adverse scenario was designed in late 2019 and was published in February 2020. While the 2020 DFAST did not adapt the severely stress scenario to incorporate the Covid-19 crisis, it disclosed additional information about predicted aggregate losses in the banking sector based on a sensitivity analysis viz-a-viz the Covid-19 crisis. Bank-level results from this exercise were not disclosed.

performance and the size of the SCB.

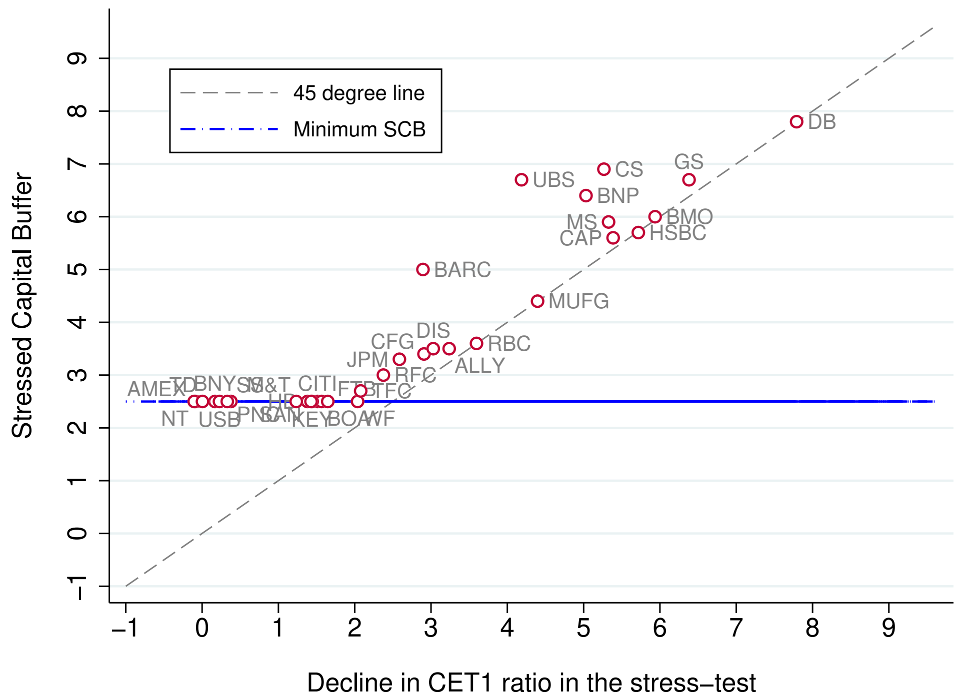

Figure 11: A comparison of the decline in CET1 ratio in the 2020 DFAST and the Stressed Capital Buffer (SCB) imposed on banks. Unit of both axes is percentage points.

A comparison of banks' CET1 ratios in the 2020 US stress-test with their actual ratios in the Covid-19 crisis can help us learn something about the noise inherent in stress testing.[40] While several factors make this comparison useful, there are caveats too.

For one, several key macroeconomic indicators (such as GDP, employment, and stock prices) in the Covid-19 crisis line up with the 2020 stress-test scenario.[41] Yet, some indicators such as the house price index do not. While the differences can render the comparison of 'a' bank's performance less meaningful, broad concordance in banks' relative performances in the test and in the current crisis is to be expected (also see Acharya et al. [2014], for instance).

[40]Older stress-tests cannot be appraised against banks' actual performances in the Covid-19 crisis as banks are likely to act on the test results and evolve materially in the meantime.

[41]The U.S. economy contracted by close to 30% (YoY) in Q2 2020; the peak unemployment rate was 15%; and the Dow Jones Index plunged by close to 30% in March 2020.

Second, the Covid-19 shock was completely unexpected, like in the case of stress-tests where the hypothetical scenarios are not known to banks in advance.

Third, capital ratios comprise of forward looking elements and thus reflect how banks would eventually perform in the crisis. Indeed, risk-weighted assets, which are a key component of banks' capital ratio calculation, are meant to incorporate banks assessment of potential future losses on (and thus riskiness of) its assets. This helps address, at least partially, a claim that the CET1 ratio of banks may not, so far, reflect fully the impact of the Covid-19 crisis. Relatedly, while extraordinary support provided to banks and to the broader economy could have dented the impact of the crisis on banks, it is unlikely that such support was targeted towards some banks and thus distorted the relative performance of banks.

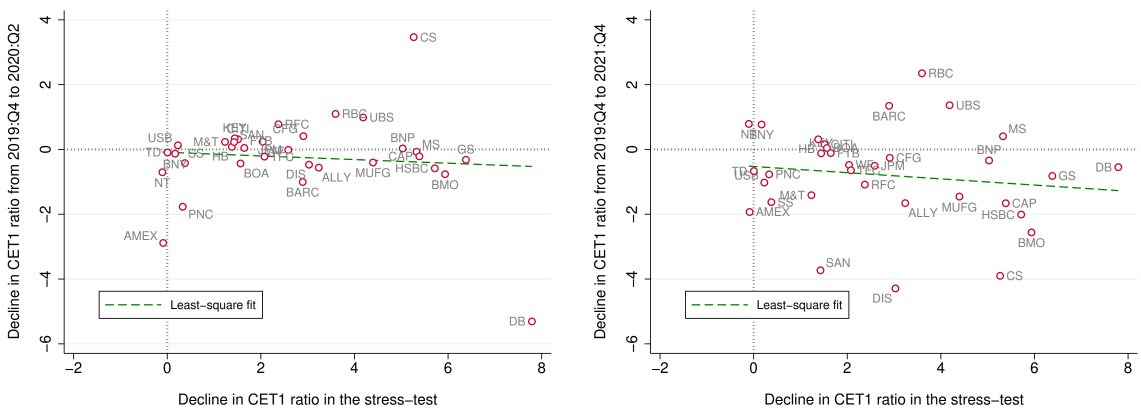

Figure 12: A comparison of the decline in CET1 ratio in the stress-test and the actual decline observed between Q4:2019 and Q2:2020 (left-hand panel) and between Q4:2019 and Q4:2021 (right-hand panel). Unit of both axes is percentage points.

To this end we compare the stress-test implied and actual decline in CET1 ratios of banks as of Q2 2020 (relative to end-2019 values) to gauge the immediate impact of the pandemic led crisis, and as of Q4 2021 to gauge the longer term impact (which is more in line with the forecast horizon of a stress-test). To complement, we pursue a similar set of comparisons in terms of CDS spreads, which provides a market-based assessment of banks' resilience. The focus is not on bank-by-bank but on an overall comparison.

In Figure 12, we document that the cross-sectional variance in test-driven changes in CET1 ratios is higher than that in case of observed changes, and that the two do not correlate at all. Moreover, while the ratios declined for almost all banks in the test (points to the right of the vertical zero line), it rose for many in reality (points below the horizontal zero line). In the case of Deutsche Bank USA, for instance, the CET1 ratio declined by 8 percentage points (pp) in the test, while during H1 2020, the same ratio rose by 5 pp. The said ratio still stands above its Q4 2019 value. We observe similar discordances in the case of CDS spreads (see Figure 13). Taken together, the higher variance in test results and its lack of correlation with observed outcomes is suggestive of the noise inherent in supervisory risk-assessments.

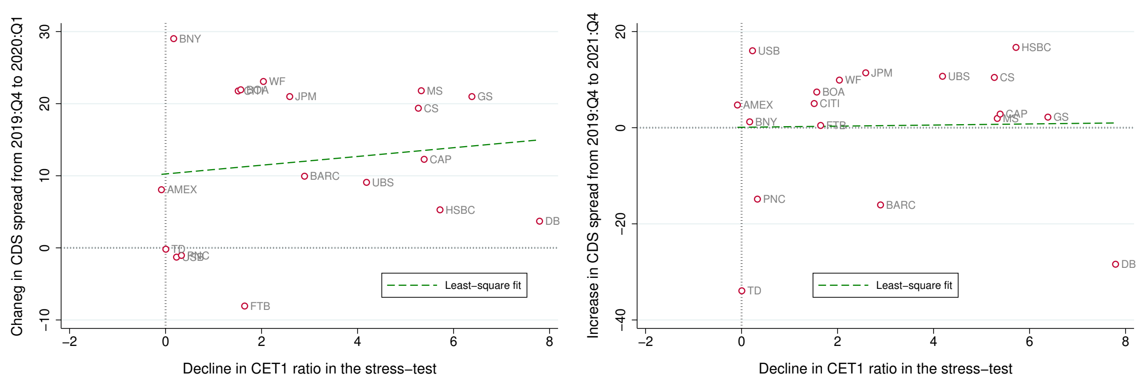

Figure 13: A comparison of the decline in CET1 ratio in the stress-test and the increase in CDS spreads in between Q4 2019 and Q1 2020 (left hand panel) and between Q4 2019 and Q4 2021 (right-hand panel). CDS data is not available for all banks in the sample.

7 Conclusion

Use of supervisory risk assessments to determine bank-specific regulation has become an important banking oversight tool for policymakers since the great financial crisis of 2008. Such assessments have helped supervisors and regulators in better gauging banks' idiosyncratic risks and in bolstering financial stability. Supervision continues to evolve and improve based on the lessons learnt over the years. Despite these enhancements, risk assessments continues to be noisy, not least due to fundamental difficulties inherent in identifying risks.

We build a model to assess the trade-offs in making supervisory assessments more accurate and draw insights for the jointly optimal supervision and regulation. A key insight from the model is that noisy supervision not only reduces welfare directly due to mis-classification of banks and misdirected capital requirements, but also by creating adverse ex-ante incentives. Going against the conventional wisdom, we show that in the presence of information frictions, higher capital requirements may lead to more risky banks. As such, the calibration of regulation on the basis of an assessment should be inversely related supervisory accuracy. In the extreme case of very low accuracy, capital regulation should be insensitive to supervision. Furthermore, when bank defaults are more costly, the supervisor must focus on reducing the rate at which riskier banks go unidentified as opposed to lowering the chances that a healthy bank is penalised. Relatedly, when incurring a cost to improve assessment accuracy is possible, the supervisor should improve accuracy and make it generally easier for banks to pass the assessment while the regulator should concurrently reduce the surcharge for banks that fail the assessment.

The overarching takeaway of the paper, therefore, is that supervision and regulation go hand-in-hand. It is important to design and optimise jointly these two aspects of a banking oversight regime to avoid adverse effects and maximise welfare.

The parsimony and tractability of our model makes it amenable to extensions of interest. For instance, some regulators have discussed maintaining a surprise element in supervisory assessments such as stress-tests on the grounds that it can help avoid pre-positioning or complacency by banks.[42] The welfare effects of a surprise element is not obvious because while it can limit the scope for gaming by banks, higher regulatory uncertainty can weaken the link between the effort banks exert and their performance in the test. This can make banks exert less effort towards improving their risk-adjusted return.

[42] See, for instance, the remarks by Mr Jerome H Powell, Chair of US Federal Reserve System, at the research conference titled "Stress Testing: A Discussion and Review" on 9 July 2019. In fact, the continuous evolution of the stress-test regime may be motivated by this pursuit.

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Appendix

A Competitive equilibrium is inefficient

We compare the dates 1 and 2 competitive equilibrium allocations with those of a benevolent social planner. For now we ignore the banker's date-0 problem, ie it's effort choice, and return to this consideration in later part of the paper. Without loss of generality, we refer to an $s - type$ banker, where s could be either high or low.

We consider a constrained social planner who maximizes the date-1 and date-2 equally weighted welfare of the household and the banker by choosing the level of deposit funding on behalf of the banker, taking as given the household's first order condition:

$$\max_{d} \quad c_1 + \beta \mathbb{E}(c_2 + n) \quad s.t. \quad R = 1/\beta; \quad c_1 = \bar{Y} - d; \quad c_2 = \bar{Y} + Rd - T$$

Recall that the banker does not consume on date-1, and note that $c_2 + n$ denotes the combined consumption of the household and the banker on date-2. Since the planner internalises the effect of choosing d on n and T, we can solve for $c_2 + n$ using expressions for n and T from equations (3) and (4) respectively:

$$c_2 + n = \bar{Y} + \psi g(k+d) - \Delta\psi g(k+d)\mathbb{1}\left(\psi \leq \frac{Rd}{g(k+d)}\right) \tag{13}$$

Next, we rewrite the planner's objective after plugging in the expressions for c_1, c_2, n, rearranging terms using the household's FOC, and segregating the expectation (i.e. the integral on $c_2 + n$) at the ψ cutoff that determines bank default:

$$\max_d \quad (1+\beta)\bar{Y} + \underbrace{\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f_s(\psi) d\psi}_{\text{Banker's date-1 objective}} + \tag{14}$$

$$\beta \int_0^{\frac{Rd}{g(k+d)}} \Big(\psi g(k+d) - Rd - \Delta \psi g(k+d)\Big) f_s(\psi) d\psi. \tag{15}$$

Segregating the integral into two parts allows to clearly see that the first part matches the bank's objective function, and facilitates a comparison of bank's and planner's problems. The planner's FOC is as follows:

$$\begin{aligned} 0 \;=\; & \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi \;+ \\ & \underbrace{\beta \int_0^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d)(1-\Delta) - R\Big) f_s(\psi) d\psi - \beta \Delta \psi g(k+d) \frac{\partial \frac{Rd}{g(k+d)}}{\partial d} f_s\left(\frac{Rd}{g(k+d)}\right)}_{\text{Bank-failure inefficiency}} \end{aligned} \tag{16}$$

Equation (16) uncovers a wedge between the planner's FOC and the bank's date-1 FOC in the unregulated economy i.e. equation (7) with $\Lambda_s = 0$. We refer to this wedge as the bank-failure inefficiency.

Next, we study what this inefficiency means for the bank's capital ratio choice in the competitive equilibrium. Assume that the inefficiency term is positive. Then, since $g(.)$ is concave:

$$\frac{\partial \frac{Rd}{g(k+d)}}{\partial d} = \frac{R\Big(g(k+d) - dg'(k+d)\Big)}{g(k+d)^2} > 0.$$

In turn, this means that

$$\beta \int_0^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi > \beta \int_0^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d)(1-\Delta) - R\Big) f_s(\psi) d\psi > 0$$

Then, $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi$ must also be positive since the integrand is increasing in ψ. But this is a contradiction since the overall expression for the planner's FOC must equal zero, which means both terms cannot be positive. In turn, this means that the inefficiency term must be negative, since it must be smaller than the other term in the planner's FOC, and that $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi > 0$. Next, we know from the bank's date-1 FOC that the choice of level of deposits in the unregulated competitive equilibrium, say d^{CE}, satisfies $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi = 0$. But since $g(.)$ is concave, it must be that $d^{CE} > d^*$ where d^* solves the constrained planner's FOC.

It is worthwhile to stress that the bank-failure inefficiency exists even when bank default is not costly ie $\Delta = 0$. Bankruptcy cost in fact amplifies this inefficiency. To see this, note from the discussion above that the inefficiency term in the planner's FOC – which is negative – is decreasing in Δ. As such, the inefficiency term in equation (16) becomes larger in magnitude as Δ increases.

B Implementing the constrained planner's solution

The goal of the regulator is to set a minimum capital-ratio requirement $k/d \geq \chi$ such that it maximizes welfare. In this sense, the regulator's decision problem is very similar to that of a constrained planner. While a planner chooses deposits on behalf of the bank to maximise welfare, a regulator imposes a minimum capital-ratio requirement, which in turns entails a given level of deposits when capital is fixed and the requirement is binding. This is formally seen by comparing equations (7) and (16). Indeed, the first terms are identical. And to the extent the Lagrange multiplier Λ (ie shadow cost) on the regulatory constraint in (7) is equal to the bank-failure inefficiency term in (16), the solution to the two equations is identical. It then also follows that when bankruptcy cost is higher – which leads to a greater inefficiency as we showed in Lemma 2 – it rationalises a higher minimum capital-ratio requirement.

C Optimal regulation: Full information

Consider the non dis-aggregated version of the planner's date-1 FOC – i.e. equation (16) – for both high- and low-type banks. This characterises the optimal level of deposits in each case.

$$0 = \int_0^\infty \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi = \mu_s g'(k+d) - R \quad s \in \{H, L\} \tag{17}$$

The total derivative of d with respect μ_s implies:

$$g'(k+d) + \mu_s g''(k+d)\frac{\partial d}{\partial \mu_s} = 0 \implies \frac{\partial d}{\partial \mu_s} > 0 \quad s \in \{H, L\} \tag{18}$$

This immediately implies that the optimal d is higher, or equivalently, the optimal χ is lower for a high-type bank.

D No surcharge if stress-test accuracy is low

Welfare as a function of the surcharge x can be written based on the regulator's problem as follows (note that e also depends on x in this expression):

$$\max_x \quad W(x) = \beta p(e)\Big(q_H U_H(\chi^o) + (1-q_H)U_H(\chi^o + x)\Big) +$$

$$\beta(1-p(e))\Big(q_L U_L(\chi^o) + (1-q_L)U_L(\chi^o + x)\Big) - \zeta(e)$$

Our goal is to identify 'a' non-trivial set of (q_H, q_L) where $W(0) > W(x) \; \forall \; x > 0$, i.e. a zero surcharge is optimal.[43] A sufficient condition for this to be the case is $W'(x) < 0 \; \forall \; x > 0$.

[43] Our goal is to not fully characterise the set of (q_H, q_L) for which the optimal surcharge is zero. We only wish to show that with low-enough accuracy, imposing a surcharge is sub-optimal.

To this end, we consider the first-order condition of the regulator's problem:

$$\frac{dW}{dx} = p'(e)e'(x)\Big(q_H U_H(\chi^o) + (1-q_H)U_H(\chi^o + x)\Big) + p(e)(1-q_H)U'_H(\chi^o + x) -$$

$$p'(e)e'(x)\Big(q_L U_L(\chi^o) + (1-q_L)U_L(\chi^o + x)\Big) + (1-p(e))(1-q_L)U'_L(\chi^o + x) - \zeta'(e)e'(x)$$

To characterise the sign of this expression, we make a few assumptions, again with the goal to find *sufficient* conditions under which the optimal surcharge is zero.

- First we assume that $x \in [0, \chi^o_L - \chi^o]$. The upper bound corresponds to a surcharge amount that results in a requirement for the low-type banks that is equal to the ex-post optimal requirement χ^o_L. In principle, the optimal surcharge could be higher (due to its effect on improving ex-ante effort), but that would entail a welfare decreasing effect in case of both high- and low-type banks.

- Second, we assume that (q_H, q_L) are such that the effort exerted by the bank decreases as surcharge increases (as per Lemma 6).

Next, since $U_s(\chi^o + x), s \in \{L, H\}$ is a concave function of x, $\chi^o_L \geq \chi^o \geq \chi^o_H$ implies the following: (i) $U_H(\chi^o) \geq U_H(\chi^o + x)$; (ii) $U'_H(\chi^o + x) \leq 0$; (iii) $U_L(\chi^o) \leq U_L(\chi^o + x)$; and (iv) $U'_L(\chi^o + x) \geq 0$; $\forall\, x \in [0, \chi^o_L - \chi^o]$. It then follows that:

$$\frac{dW}{dx} \leq p'(e)e'(x)U_H(\chi^o) + p(e)(1-q_H)U'_H(\chi^o + x) - p'(e)e'(x)U_L(\chi^o) +$$

$$(1-p(e))(1-q_L)U'_L(\chi^o) - \zeta'(e)e'(x)$$

Finally, we re-arrange and set the right-hand-side expression to zero:

$$p(e)U'_H(\chi^o + x) + (1-p(e))U'_L(\chi^o) - p(e)q_H U'_H(\chi^o + x) - (1-p(e))q_L U'_L(\chi^o) +$$

$$\underbrace{p'(e)e'(x)\Big(U_H(\mathcal{X}^o) - U_L(\mathcal{X}^o)\Big)}_{A<0} - \zeta'(e)e'(x) = 0$$

$$\implies \underbrace{\frac{A}{p(e)U'_H(\mathcal{X}^o + x)} + 1 + \frac{(1-p(e))U'_L(\mathcal{X}^o)}{p(e)U'_H(\mathcal{X}^o + x)} - \zeta'(e)e'(x)}_{\tau_0 <> 0} - q_L \underbrace{\frac{(1-p(e))U'_L(\mathcal{X}^o)}{p(e)U'_H(\mathcal{X}^o + x)}}_{\tau_1 < 0} = q_H$$

$$\implies q_H = \tau_0 - \tau_1 q_L \tag{19}$$

In equation (19), while the slope is positive, the intercept can be positive or negative, depending on the underlying parameters. The equation implies that when $q_H < \tau_0 - \tau_1 q_L$ the surcharge should be zero, as also indicated in Figure 2.

E Optimal surcharge with bankruptcy costs

The regulator's problem is as follows:

$$\max_x \quad W(x) = \beta p\Big(q_H U_H(\mathcal{X}^o, \Delta) + (1-q_H)U_H(\mathcal{X}^o + x, \Delta)\Big)$$

$$\beta(1-p)\Big(q_L U_L(\mathcal{X}^o, \Delta) + (1-q_L)U_L(\mathcal{X}^o + x, \Delta)\Big)$$

Here Δ in the utility function U formally expresses the dependence of welfare on failure costs. The attendant first-order condition is as follows, where the D_i operator indicates the derivative with respect to the i^{th} argument of U:

$$p(1-q_H)D_1U_H(\mathcal{X}^o + x, \Delta) + (1-p)(1-q_L)D_1U_L(\mathcal{X}^o + x, \Delta) = 0$$

Next, we take the total derivative of this expression with respect to Δ:

$$p(1-q_H)\Big(D_{11}U_H(\mathcal{X}^o + x, \Delta)\frac{dx}{d\Delta} + D_{12}U_H(\mathcal{X}^o + x, \Delta)\Big) +$$

$$(1-p)(1-q_L)\Big(D_{11}U_L(\mathcal{X}^o + x, \Delta)\frac{dx}{d\Delta} + D_{12}U_L(\mathcal{X}^o + x, \Delta)\Big) = 0$$

$$\implies -\underbrace{\Big(p(1-q_H)D_{11}U_H(\mathcal{X}^o+x,\Delta)+(1-p)(1-q_L)D_{11}U_L(\mathcal{X}^o+x,\Delta)\Big)}_{A}\frac{dx}{d\Delta}=$$

$$p(1-q_H)D_{12}U_H(\mathcal{X}^o+x,\Delta)+(1-p)(1-q_L)D_{12}U_L(\mathcal{X}^o+x,\Delta)$$

Since both U_H and U_L are concave functions of x, $A<0$. To sign the RHS, consider $U_s, s=\{H,L\}$:

$$U_s(\mathcal{X}^o+x,\Delta)=\bar{Y}-d+\beta g(k+d)(\mu_s-\Delta\int_0^{\frac{Rd}{g(k+d)}}\psi f_s(\psi)d\psi) \quad where \quad d=\frac{k}{\mathcal{X}^o+x}$$

$$\implies D_2U_s(\mathcal{X}^o+x,\Delta)=-\beta g(k+d)\int_0^{\frac{Rd}{g(k+d)}}\psi f_s(\psi)d\psi)$$

$$\implies D_{21}U_s(\mathcal{X}^o+x,\Delta)=-\beta\Bigg(g'(k+d)\frac{dd}{dx}\int_0^{\frac{Rd}{g(k+d)}}\psi f_s(\psi)d\psi+$$

$$g(k+d)\frac{d}{dd}\left[\frac{Rd}{g(k+d)}\right]\frac{Rd}{g(k+d)}f_s\left(\frac{Rd}{g(k+d)}\right)\frac{dd}{dx}\Bigg)$$

As x increases, d decreases i.e. $\frac{dd}{dx}<0$. Also, as d increases, the upper limit on the integral is increases (recall $g(.)$ is concave), which means that by application of Leibniz rule, $D_{21}U_s(\mathcal{X}^o+x,\Delta)>0$. Since U is a continuous function in both its arguments, $D_{21}U_s(\mathcal{X}^o+x,\Delta)=D_{12}U_s(\mathcal{X}^o+x,\Delta)>0$ for both $s=H,L$.

F Moving along the ROC

We are interested in how the expression below responds to an increase in ρ:

$$\frac{U_H(\mathcal{X}^o)-U_H(\mathcal{X}^o+x)}{U_L(\mathcal{X}^o)-U_L(\mathcal{X}^o+x)}$$

Consider ρ to be the bankruptcy cost Δ (the proof in case ρ is the riskiness of assets σ follows a similar strategy). Then, for $s \in \{H, L\}$, $U_s(\mathcal{X}^o) - U_s(\mathcal{X}^o + x)$ equals:

$$\beta g(k+d(\mathcal{X}^o))(\mu_s - \Delta \int_0^{\frac{Rd(\mathcal{X}^o)}{g(k+d(\mathcal{X}^o))}} \psi f_s(\psi) d\psi) - \beta g(k+d(\mathcal{X}^o+x))(\mu_s - \Delta \int_0^{\frac{Rd(\mathcal{X}^o+x)}{g(k+d(\mathcal{X}^o+x))}} \psi f_s(\psi) d\psi)$$

where $d(y) = \frac{k}{y}$. The derivative of this expression w.r.t. Δ leads to the following:

$$\beta\Big(g(k+d(\mathcal{X}^o+x)) \int_0^{\frac{Rd(\mathcal{X}^o+x)}{g(k+d(\mathcal{X}^o+x))}} \psi f_s(\psi) d\psi) - g(k+d(\mathcal{X}^o)) \int_0^{\frac{Rd(\mathcal{X}^o)}{g(k+d(\mathcal{X}^o))}} \psi f_s(\psi) d\psi)\Big)$$

Next, note that $d(\mathcal{X}^o + x) < d(\mathcal{X}^o)$. Also note that $\frac{Rd(\mathcal{X}^o+x)}{g(k+d(\mathcal{X}^o+x))} < \frac{Rd(\mathcal{X}^o)}{g(k+d(\mathcal{X}^o))}$ as $g()$ is concave. This implies that the term inside the bracket is negative, i.e. $U_s(\mathcal{X}^o) - U_s(\mathcal{X}^o + x)$ decreases as Δ increases.

Next recall that $U_s(\mathcal{X}^o) - U_s(\mathcal{X}^o + x)$ is negative for low type banks but it is positive in the case of high type banks (a surcharge is desirable in case of low-type banks but not in case of high-type banks).

All together, this means that a higher Δ would make $U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)$ smaller but still positive and $U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x)$ further more negative. The ratio of the two would remain negative and would become smaller in absolute value, or in other words, would increase. This implies then, that, $\frac{d\nu}{d\Delta} > 0$.

G Increasing the area under the ROC

The problem of the regulator is as follows, where we continue to assume that the probability of a given type of bank is fixed:

$$\max_{x,\nu,\theta} \quad \beta p\Big(q_H(\nu,\theta)U_H(\mathcal{X}^o) + (1 - q_H(\nu,\theta))U_H(\mathcal{X}^o + x)\Big) +$$

$$\beta(1-p)\Big(q_L(\nu,\theta)U_L(\mathcal{X}^o) + (1 - q_L(\nu,\theta))U_L(\mathcal{X}^o + x)\Big) - C(\theta)$$

Without loss of generality, we let $q_H(\nu,\theta) = 1-\nu$ and let $q_L(\nu,\theta)$ be decreasing in both arguments. These specifications follow from the economic interpretations of ν and θ: an increase in ν would generally increase the failure rate of banks in the stress-test, leading to a lower q_L (desirable) but also a lower q_H (undesirable). An increase in θ can be thought of as moving the ROC curve ”upwards” for each ν, which entails a lower q_L while q_H remains fixed. In addition, to capture the fact that decreasing the false negative rate becomes increasingly more difficult, we let also $q_L(\nu,\theta)$ be convex in both arguments. Finally, for analytical tractability, we assume that $\mathcal{C}(\theta) = C\theta$ where C is the linear cost of accuracy.[44] The first order conditions (FOCs) are as follows:

$$[x]: \quad p\nu\frac{dU_H(\mathcal{X}^o+x)}{dx} + (1-p)(1-q_L(\nu,\theta))\frac{dU_L(\mathcal{X}^o+x)}{dx} = 0$$

$$[\nu]: \quad \frac{dq_L(\nu,\theta)}{d\nu} = \left(\frac{p}{1-p}\right)\frac{U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o+x)}{U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o+x)}$$

$$[\theta]: \quad \beta(1-p)\underbrace{\frac{dq_L(\nu,\theta)}{d\theta}}_{<0}\underbrace{\Big(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o+x)\Big)}_{<0} - C = 0$$

While obtaining the comparative statics in general is not possible, a sequential approach delivers useful insights that we also verify numerically. First, assuming ν and x stay put in response to a change in C, the derivative of the FOC for θ w.r.t. C implies that:

$$\beta(1-p)\underbrace{\frac{d^2q_L(\nu,\theta)}{d\theta^2}}_{>0}\frac{d\theta}{dC}\Big(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o+x)\Big) - 1 = 0$$

This leads to $\frac{d\theta}{dC} < 0$, i.e. as the cost of accuracy decreases, the regulator would choose an ROC curve with a higher area under it.

Second, assuming x stays put, and assuming $\frac{d\theta}{dC} < 0$, we obtain the optimal response

[44] We are interested in comparative statics w.r.t. to a scalar parameter that governs the cost of accuracy. A more general functional form such as $\mathcal{C}(\theta) = Cf(\theta)$ where $f(.)$ is non-trivial would also work for the proof.

in ν. The derivative of the FOC for x w.r.t. C implies that:

$$\implies p \underbrace{\frac{dU_H(\chi^o + x)}{dx}}_{<0} \frac{d\nu}{dC} = (1-p) \underbrace{\frac{dU_L(\chi^o + x)}{dx}}_{>0} \underbrace{\left[\underbrace{\frac{dq_L(\nu,\theta)}{d\nu}}_{<0} \frac{d\nu}{dC} + \underbrace{\frac{dq_L(\nu,\theta)}{d\theta}}_{<0} \underbrace{\frac{d\theta}{dC}}_{<0} \right]}_{\frac{dq_L(\nu,\theta)}{dC}}$$

This implies that $\frac{d\nu}{dC}$ and $\frac{dq_L}{dC}$ are of opposite signs, i.e. following an increase in C, ν and q_L respond in opposite directions. Moreover, assuming that $\frac{d\theta}{dC} < 0$ continues to hold, $\frac{d\nu}{dC}$ cannot be negative, as otherwise we arrive at a contradiction. So $\frac{d\nu}{dC} > 0$ and $\frac{dq_L}{dC} < 0$

Third, to gauge the response in surcharge x, we consider the total derivative of the FOC for x w.r.t. C:

$$p\frac{dU_H(\chi^o + x)}{dx}\frac{d\nu}{dC} + p\nu\frac{d^2U_H(\chi^o + x)}{dx^2}\frac{dx}{dC} - (1-p)\frac{dU_L(\chi^o + x)}{dx}\frac{dq_L(\nu,\theta)}{dC} +$$

$$(1-p)(1-q_L(\nu,\theta))\frac{d^2U_L(\chi^o + x)}{dx^2}\frac{dx}{dC} = 0$$

$$\implies sign\left(\frac{dx}{dC}\right) = sign\left(p\frac{dU_H(\chi^o + x)}{dx}\frac{d\nu}{dC} - (1-p)\frac{dU_L(\chi^o + x)}{dx}\frac{dq_L(\nu,\theta)}{dC}\right)$$

That is, while the direction of response in x is not obvious and depends on a range of model primitives, it is more likely to be negative when its marginal increase in welfare due to a surcharge in case of a low-type bank is smaller, or when the marginal decrease in welfare in case of high-type bank is bigger.